RESEARCH ON SYNERGY CAUSED BY NETWORK EXTERNALITY: THE CASE OF LITHUANIAN FIXED TELEPHONE COMMUNICATIONS MARKET

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Abstract. The paper presents a theoretical model of synergetic effect and explores the Lithuanian fixed telephone communications market before and after liberalization in accordance with network externality and user utility change when new users login. Based on the developed theoretical model, an empirical research was done. The aim of the research was to investigate the synergetic effect and to show the importance of network externality in the Lithuanian fixed telephone communications market. According to our theoretical model, there may be a situation when, after liberalization and due to increased competition, the profit of the market monopolist (ex) grows. This means that the competition caused by the decrease in revenue is offset by gains in the supply. In this case, the synergetic effect caused by the network externality takes place.

Key words: synergetic effect, network externality, oligopoly

Introduction

To increase the competitiveness of Lithuania’s economic performance, its markets were liberalized. This wave did not spare the communication business. For a successful operation in an integrated market in which it is necessary to bear in mind the fact that added value creation and increase depend on the ability to make good strategic management decisions which lead to economic benefits, i.e. synergies. In economic literature, the synergetic effect is analysed in different aspects. One of them is the view that this added value is important in the social context.

In this work, the term of synergetic effect is understood in a broad sense. It covers not only the activities of a separate economic agent, but also the impact on their groups. For example, the merging companies do not gain additional economic benefit (synergetic effect), but from the social point of view it could be useful, i.e. their utility grows. Benefit may occur through increased competition, growth of supply, etc. The classical economic theory suggests that liberalization reduces monopolist profit. The hypothesis of this study is as follows: after liberalization of the Lithuanian fixed telephone communications market, the profit of the monopolist TEO LT could be positively affected.

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The article states that there may be a situation when network externality occurs, when competition is useful for a monopolist. In this case, a synergetic effect occurs. This phenomenon in the context of the Lithuanian telecommunication market was not analysed. The developed model can be applied to other kinds of markets where network effects occur. This is what makes this study actual and novel.

One knows that the value of goods and services grows with increasing the number of people using them. There are the products that are used in conjunction with other products at the same time. The latter, when used alone, are almost worthless. The buyers that use these products make up networks. In these networks user utility grows by logging in new users. A market characterized by this feature is called a network market, and it has a positive effect on consumption by network externality. The first economists who started to analyse these impacts were Jeffrey Rohlfs (1974), Shmuel Oren and Stephen Smith (1981), Michael Katz and Carl Shapiro (1985), Joseph Farrell and Garth Saloner (1985). Later works were extended by Nicholas Economides and Charles Himmelberg (1994), Stan J. Liebowitz and Stephen E. Margolis (1994) and others.

The goal of the present study was to identify and measure the synergetic effect of the Lithuanian fixed telephone communications market. To attain this goal, the following tasks were raised:

1. By analysing the economic literature, to develop a theoretical model and to accomplish an empirical study.
2. To investigate the possibilities of positive effects on monopolist profit after liberalization.

The object of investigation was the Lithuanian fixed telephone communications market. Mathematical statistical methods were used in this research. Regression analysis was made by using the ordinary least squares method, and the problem of optimisation was solved by using differentiation calculus.

**Characteristics of network externality**

Network externality is the situation when consumer utility by good consumption increases with the number of other agents consuming the good (Katz and Shapiro, 1985). The more general definition was proposed by J. Liebowitz and S. Margolis (1995). They state that network externality is the increase of the net value of an action, resulting from the growth of the same number of users performing this action. In the economic literature, generally considered products where network externality can occur are fax machines, telecommunication, credit cards, computer hardware and software, etc.

There are two types of network externality: direct and indirect (Katz and Shapiro, 1985; Economides, 1996). The direct network externality is generated by consumption of the same product. The most typical example of direct externality is telecommunication networks such as the Internet, phone lines, etc.
The indirect network externality occurs when product value is added by the growing number of substitutions. A good example of this kind is computers and its software. This effect is often called the hardware–software paradigm (Katz and Shapiro, 1994).

Upon examining the vast economic literature, three sources of network externality may be pointed out:
- consumer expectations;
- consumer and manufacturer coordination;
- product substitution.

Expectations play a key role in the process of product choice. That is why consumer utility depends on the number of purchasing customers. This phenomenon was analysed by Ohashi (2003) in the study of the VCR market in the 1980s.

Talking about the second network externality factor – coordination – it is possible to prove that it takes different directions in the context of consumer and producer interests. The first ones always seek the unity and coherence, while the latter often do the opposite. Coordination of producer profits exerts an influence in both directions. The consequence of the compatibility of the products or a unified standard means the market increase, but at the same time the growth of competition. The market increase leads to the growth of revenue, i.e. to a synergetic effect, whereas the competition causes price decline.

A significant impact on the strength of the network effect is made by the so-called network switching costs. These costs incur users or producers who want to switch to another network. Burnham, Frels and Mahajan (2003) identify eight types of switching costs: economic risk (uncertainty associated with future), evolution (associated with search of new relations, and communication analysis), learning (associated with the use of a product and new skills acquisition), setup (associated with a new product setup), benefit loss (related with customer discount loss), monetary loss (associated with agreement termination costs), personal relationship loss and brand relationship loss.

The third source of network externality is substitution. It is defined as the quality of services provided by sale dependence on network size. A good example is home appliances. Consumers prefer the brand whose network of sales and after-sales is better developed. Katz and Shapiro (1985) have analysed the substitution as a source of network externality and revealed that it is dependent on the following factors:
- information is more easily available for more popular brands,
- the role of market share as a signal of product quality,
- psychological band-wagon effects.

A very important characteristic of the market with network externality is critical mass. The concept used in physics was borrowed by Rohlfs (1974) and Oren with Smith (1981). They used this concept in the analysis of the telecommunication market. Critical mass is the minimal non-zero equilibrium market size that leads to a significant increase in network expansion where network externality exists (Economides, 1996). Economides and Himmelberg (1995) relate the term of critical mass with the “chicken-and-egg” para-
dox. Consumers are not interested in purchasing the good because the installed base is too small, and the installed base is too small because a too small number of consumers have purchased the good.

There is some instability in the markets when network externality occurs. Bensen and Farrell (1994) called this characteristic tippy. This instability can be explained by several reasons. First, there is not the only equilibrium (Table 1), because in the reverse form, the demand function is the second degree of quantity. Second, instability occurs when the network size $X_t$ is influenced by the network sizes $X_{t-i}$ that existed in the past:

$$X_t = f(X_{t-1}, X_{t-2}, ..., X_{t-n}),$$  \hspace{1cm} (1)

where $t$ is time moment, and $n$ is the number of historical periods that have influenced the current network size. This property in the economic literature is called path dependence. This term in economics corresponds in physics to the term of hysteresis (it refers to the systems that have a memory, in which the effects of the current input (or stimulus) on the system are experienced with a certain delay in time). This process leads to market inefficiencies in the economy, as the initial conditions cause the consequences that are very expensive to replace. David (1985) and Arthur (1989) were the first to analyze this issue. David (2000) defines path dependence as a stochastic process whose evolution is dependent on the historical process, i.e. the demand of goods characterized by network externalities depends on the decisions of buyers that were made earlier.

**Model**

Let us analyse the reverse demand functions with network externality:

$$P = f(X, T),$$  \hspace{1cm} (2)

where $P$ is price, $X$ is quantity (in the Lithuanian fixed telephone communications market $X$ is the number of lines), and $T$ is the network effect. Many different functional forms are used in economic literature. For example, Economides and Himmelberg (1995a, 1995b) use the following form:

$$P(n, n_e) = \frac{k + \delta \cdot f(n_e)}{G(1 - n)},$$  \hspace{1cm} (3)

where $n$ is the normalized current network size ($0 \leq n \leq 1$), $n_e$ is the expected network size, $\delta$ and $f$ are the network externality effect coefficient (0 or 1) and function, $G$ denotes the inverse function of consumer distribution normalized in the interval of $[0,1]$, and its argument is the residual network part. If $\delta$ equals zero, the network externality does not occur. This is a simple demand function. If $\delta$ equals one, then the market affects the impact of network externality.

Rohlfs (1974) and Oren with Smith (1981) present another reverse demand function:

$$P(X) = t \cdot X - a \cdot X^2 + b,$$  \hspace{1cm} (4)
where \( t \) is the network externality coefficient, while \( a \) and \( b \) are the demand function coefficients. It is obvious that this is a square function.

Let us consider an oligopolistic market in which the network externality occurs and the demand function is described by equation (4). Suppose that there are \( n \) firms in the market, one of them being considered the dominant while the others are followers. Then we get the equilibrium quantity:

\[
X = (n - 1) \cdot x_s + x_L.
\]

(5)

In this paper, it is assumed that the followers offer the same supply. Let’s assume that the companies’ cost function \( TC \) is described as:

\[
TC = k_1 \cdot x_2 + k_2 \cdot x + k_3,
\]

(6)

where \( x \) is the quantity of goods or services, and \( k_i \) are the coefficients reflecting the technology. Let us adopt the Stackelberg leadership model. A firm follower will seek to maximize the profit by taking the quantity offered by the leader and the number of firms \( n \) as fixed parameters:

\[
\frac{d(P(x_s, x_L, n) \cdot x_s - TC_S(x_s))}{dx_s} = 0.
\]

(7)

From (7) we find the reaction function \( x_s = f(x_L) \) of the followers. The leader knowing the reaction function of the followers will seek to maximize profits with respect to quantity \( x_L \) and the number of firms \( n \):

\[
\frac{d(P(x_s, x_L, n) \cdot x_L - TC_L(x_L))}{dx_L} = 0,
\]

(8)

\[
\frac{d(P(x_s, x_L, n) \cdot x_L - TC_L(x_L))}{dn} = 0.
\]

(9)

It is estimated by the author of this study that for (7), (8) and (9) the result of solving the differential equations will be the same if one applies other oligopolistic models such as price leadership, simultaneous quantity or price setup.

The solution of differential equations (7), (8) and (9) is not simple and takes a lot of mathematical computation, so it is represented in Annex 1. Because of the third degree of the equations, we get three equilibriums. The obtained results are presented in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Market equilibriums</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilibrium</td>
</tr>
<tr>
<td>( x_L^* = 0; )</td>
</tr>
<tr>
<td>( n^* = f_1(a, b, k_1, k_2, t) )</td>
</tr>
</tbody>
</table>
It should be noted that the forms of functions $f_1$, $f_2$ and $f_3$ are founded, but they are irrelevant for the further analysis and are not shown here. In the solutions of equations (7), (8) and (9), it is founded that $x_S^* = x_L^*$. This is the result of the assumption that cost functions are the same for all firms. Only the second equilibrium is important for the analysis (Table 1). The leader will generate a synergetic effect (the profit will grow despite the increase in competition) if the number of firms is greater than one, i.e. $n^* > 1$:

$$4(t \cdot k_1 + a \cdot k_2 - a \cdot b) > t_2.$$  (10)

What is the meaning of this economic condition? It would be more obvious if we take that $a = 0$, $b = 0$, then $4k_1 > t$. This means that the synergetic effect will occur when the quarter of the second derivative of the cost function (representing the rate of cost growth by increasing the supply) is greater than the effect of network externality.

The synergetic effect can be expressed as an increased number of firms $\Delta S_n$ (increased competition) and as increased supply $\Delta S_x$. In the first case,

$$\Delta S_n = n^* - 1 = \frac{4 \cdot t \cdot k_1 - 4a \cdot b - t^2 + 4a \cdot k_2}{4a \cdot b + t^2 - 4a \cdot k_2}.$$  (11)

In case of $\Delta S_n > 1$, we have a positive and in case of $\Delta S_n \leq 1$ a negative synergetic effect. In the latter case, there is only one firm in the market. $\Delta S_n$ shows the number of firms that is optimal for the monopolist.

The supply synergetic effect is the following:

$$\Delta S_x = X_0 - X_m \frac{t + 2k_1 \mp 2\sqrt{t^2 - 2tk_1 + k_1^2 - 3ak_2 + 3ab}}{6a}.$$  (12)

The proof of this equation is presented in Appendix 2. $X_0$ and $X_m$ are the equilibrium supply of oligopolistic and monopolistic markets. In case of $\Delta S_x > 0$, we have a positive and in case of $\Delta S_x < 0$ a negative synergetic effect. Then, at $\Delta S_x = 0$, we have the same supply. What is the economic sense of this inequality? It must be admitted that the benefit accrues from increased supply not to the seller, but to the buyer. Therefore, the value $\Delta S_x$ indicates how much the market increases the supply because of competition.

From these calculations, the equilibrium quantity $X_p$ and price $P_p$ are

$$X_p = x_L^* \cdot n^* = \frac{t}{2 \cdot a}, P_p = \frac{t^2}{4 \cdot a} + b.$$  

It should be noted that the equilibrium quantity is equal to the demand quantity when the price exceeds its maximum: $X_p = \frac{dP}{dX}$ (Fig. 1).
Figure 1 presents the leader’s marginal costs \( \frac{dTC}{dX_L} \) and the reverse demand \( P(X) \) curves. It is clear from equation (5) that \( n = \frac{X_p}{X_L} \). This is equal to the proportion of sections B and A. By using it, we can find phase transition points (line of points \( X_{m/o} \)). There are two possible phases: monopolistic and oligopolistic.

In case of the marginal cost \( \frac{dTC}{dX_L} \) and \( X_{m/o} \), the line intersection point is below the \( P_p \) level; then, the market will be monopolistic (because the proportion of sections B and A is equal to the number of competing firms and becomes smaller than one), and in case when the intersection point is higher they are oligopolistic.

**Empirical research**

Referring to the above considerations, let us evaluate the Lithuanian fixed telephone communications market demand and confirm or reject the situation when the profit of the monopolist increases after market liberalization (i.e. the synergetic effect occurs). Prior to that, there is a brief introduction. In February 1992, the Post and Telephone networks were separated, and the state-owned “Lietuvos telekomas” enterprise was established. The company acted as a natural monopoly, and in 1998 it was privatized by the “Amber Teleholding A/S” consortium (TEO LT, 2006). Since 1 January 2003, in Lithuania the fixed telephone communications market was liberalized, which meant that the exclusive rights of the “Lietuvos telekomas” came to an end. Up till that date, the company had been the only provider of this kind of service. During the first half of 2003, twenty-five companies submitted a notice of intent to engage in the fixed telephone communications market. Twelve of them started the business (The Communications Regulatory Authority of the Republic of Lithuania, 2003).

Aggregating the data of the Communications Regulatory Authority, we can present the relation of fixed telephone communications users and the price paid by the user per month (Fig. 2). The data cover the period from 1996 to the third quarter of 2009.
A closer look at Fig. 2 allows to see the so-called hysteresis loop or, in other words, the occurrence of path dependence. It is marked by a dashed line. Actual quantities are presented as dots joined by a line. This loop can be explained by the fact that the lower part occurred when the market was relatively little, i.e. before 2003. After that, the mobile communication operators had a strong influence on the fixed telephone communications market.

From the deterministic equation (4) we can produce a stochastic one:

\[ P_t = a_1 X_t^2 + a_2 X_t + a_3 X_{t-1}^2 + a_4 Y + a_5 T + a_6 + \varepsilon_t, \]

(13)

where \( a_i \) are regression coefficients, \( X_t \) is supply at moment \( t \), \( X_{t-1} \) is supply at moment \( t-1 \), \( Y \) is income, and \( T \) is trend. The estimation of this regression is presented in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variant 1</th>
<th>Variant 2</th>
<th>Variant 3</th>
<th>Variant 4</th>
<th>Variant 5</th>
<th>Variant 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_6 )</td>
<td>–</td>
<td>231.13 (86.46)</td>
<td>128.56 (75.16)</td>
<td>165.06 (108.09)</td>
<td>185 (120.65)</td>
<td>–</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>7.434 ( 10^{-5} ) (9.8 ( 10^{-6} ))</td>
<td>-4.011 ( 10^{-4} ) (1.78 ( 10^{-4} ))</td>
<td>-2.102 ( 10^{-4} ) (1.52 ( 10^{-4} ))</td>
<td>-2.741 ( 10^{-4} ) (2.06 ( 10^{-4} ))</td>
<td>-3.103 ( 10^{-4} ) (2.28 ( 10^{-4} ))</td>
<td>4.941 ( 10^{-5} ) (1.72 ( 10^{-5} ))</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>-2.182 ( 10^{-11} ) (9.8 ( 10^{-12} ))</td>
<td>2.183 ( 10^{-10} ) (9 ( 10^{-11} ))</td>
<td>1.142 ( 10^{-10} ) (7.78 ( 10^{-11} ))</td>
<td>1.453 ( 10^{-10} ) (1.03 ( 10^{-10} ))</td>
<td>1.624 ( 10^{-10} ) (1.13 ( 10^{-10} ))</td>
<td>-1.823 ( 10^{-11} ) (7.56 ( 10^{-12} ))</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>–</td>
<td>–</td>
<td>1.752 ( 10^{-5} ) (1.2 ( 10^{-5} ))</td>
<td>1.691 ( 10^{-5} ) (1.25 ( 10^{-5} ))</td>
<td>1.571 ( 10^{-5} ) (1.33 ( 10^{-5} ))</td>
<td>2.212 ( 10^{-5} ) (1.26 ( 10^{-5} ))</td>
</tr>
<tr>
<td>( a_4 )</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>-2.602 (5.33)</td>
<td>-0.801 (6.78)</td>
<td>–</td>
</tr>
<tr>
<td>( a_5 )</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>-0.262 (0.56)</td>
<td>–</td>
</tr>
<tr>
<td>Number of observations</td>
<td>16</td>
<td>16</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.36</td>
<td>0.59</td>
<td>0.70</td>
<td>0.71</td>
<td>0.72</td>
<td>0.63</td>
</tr>
</tbody>
</table>

In Table 2, standard deviations are presented in parentheses. Data used in the calculations were collected from the Communications Regulatory Authority. The variants represent the assumptions about the coefficients. In the first one, it is assumed that \( a_6 = 0 \), \( a_3 = 0 \), \( a_4 = 0 \), \( a_5 = 0 \), in the second \( a_3 = 0 \), \( a_4 = 0 \), \( a_5 = 0 \), in the third \( a_4 = 0 \), \( a_5 = 0 \), in the forth \( a_5 = 0 \), and in the sixth \( a_6 = 0 \), \( a_4 = 0 \), \( a_5 = 0 \). Table 2 shows that the determination coefficient is not very high. The reasons are as follows:

- endogeneity of variables;
- the impact of mobile operators;
- assumption about cost functions.

The endogeneity is caused by a correlation between \( P_t \) and \( \varepsilon_t \). An attempt was made to eliminate it by using the information variables, but the second order of demand func-
tions does not allow producing the reverse form. For this reason, the same assumption was made as in Economides and Himmelberg (1995b) analysis in which the demand for fax machines is examined. The model does not evaluate the influence of mobile communication operators. This impact was very strong, especially in the second half of the first decade. The third reason is the assumption that cost functions are the same. This is not so in reality. It is confirmed by the fact that new competitors do not have their own network, they use the network of the TEO LT company.

Let us estimate the cost function of the fixed operator. Based on the deterministic equation (6), we produce a stochastic one:

\[ TC_t = k_1 \cdot x_t^2 + k_2 \cdot x_t + k_3 + k_4 T + \varepsilon_t \]  

(14)

The estimations are presented in Table 3.

**TABLE 3. Cost function estimation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variant 1</th>
<th>Variant 2</th>
<th>Variant 3</th>
<th>Variant 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_3 )</td>
<td>–104.14 (197.58)</td>
<td>96.37 (107.92)</td>
<td>–4062.54 (1012.18)</td>
<td>–3941.76 (1668.39)</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>3.764 \times 10^{-4} (1.88 \times 10^{-4})</td>
<td>–</td>
<td>8.321 \times 10^{-3} (2.21 \times 10^{-3})</td>
<td>8.112 \times 10^{-3} (3.21 \times 10^{-3})</td>
</tr>
<tr>
<td>( k_1 )</td>
<td>–</td>
<td>1.742 \times 10^{-10} (9.59 \times 10^{-11})</td>
<td>–3.911 \times 10^{-9} (9.93 \times 10^{-10})</td>
<td>–3.823 \times 10^{-9} (1.59 \times 10^{-9})</td>
</tr>
<tr>
<td>( k_4 )</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–0.762 (7.82)</td>
</tr>
<tr>
<td>Number of observations</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.36</td>
<td>0.31</td>
<td>0.82</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Like in Table 2, here variants represent the assumptions about the coefficients \( k_i \). In the first case, it is assumed that \( k_1 = 0, k_4 = 0 \); second, \( k_2 = 0, k_4 = 0 \), and third, \( k_4 = 0 \). Using the results presented in Tables 2 and 3, we can reveal the impact of liberalization of fixed telephone communications market on the profit of TEO LT. This shows the possibility of a synergetic effect (\( \Delta S_n > 1 \)). By inserting values from Tables 2 and 3 (Variant 3 where determination is high) we get:

\[ \Delta S_n = 0.129. \]

Accordingly, from equation (12) we calculate two values of \( \Delta S_x \):

\[ \Delta S_x = \{270000; 360000\}. \]

The \( \Delta S_x \) values show that because of liberalization of fixed telephone communications the number of users would increase by three hundred thousand (ceteris paribus), but this growth will lead to a decrease in profit.

Based on the theoretical model, the calculated critical mass is in the interval from 0.93 to 1.7 million users. Compared to the actual values (Fig. 2) which are in the inter-
val 0.7–1.2 million, it is obvious that the fixed telephone communications market is not substantial – the critical mass is not achieved. This means that the theoretical model does not predict the occurrence of synergetic effect, and actually it did not occur. That is why liberalization has affected the profit negatively.

Conclusions

• The border of phase transition from the monopolistic to the oligopolistic market has been determined. In the context of the Lithuanian fixed telephone communications market, the estimated equilibrium is on the monopolistic side. For this reason, the monopolist is not motivated to change the market structure into oligopolistic.

• The demand of the Lithuanian fixed telephone communications market has been estimated. The determination coefficient is 0.72. This means that 72 percent of demand variation is described by the constructed model, and only 28 percent of variation is not explained.

• The synergetic effect is measured as an increased number of competitors. The estimated value is equal to 0.129. It is less than one, suggesting that for the fixed telephone communications market it is not reasonable to have more than an one company. This means that an increase in supply does not offset a decrease in revenue caused by competition.

• The estimated synergetic effect is based on increase in quantity. There are two values: \(2.7 \times 10^5\) and \(3.6 \times 10^5\). In this context, we can reasonably state that after liberalization (ceteris paribus), supply in the fixed telephone communications market would increase by 300 thousand subscribers.

• The Lithuanian fixed telephone communications market has not reached its critical mass. According to the model, the synergetic effect did not manifest itself and actually has not occurred. This means that the profit of the monopolist decreased after liberalization.

• The hypothesis that after liberalization the profit of the monopolist could increase has not been confirmed.

REFERENCES


APPENDIX 1. Market equilibrium

By inserting expression (5) into the reverse demand function (4), the profit of a follower will be

$$\Pi_S = x_S(-a(x_L + (n-1) \cdot x_S)^2 + t(x_L + (n-1)x_S)) - k_1x_S^2 - k_2x_S - k_3.$$ 

It is maximized when $$\frac{d\Pi_S}{dx_S} = 0$$. By differentiating and solving this equation according to $$x_S$$,

$$x_S = \frac{\pm 1}{-3a(n^2-2n+1)}[(2tk_1 + k_1^2 + 2ax_Lnt + 4ax_Lnk_1 + 3ab + 3ak_2 + 3an^2b - 3an^2k_2 - 6anb + 6ank_2 + a^2x_L^2 - 4k_1ax_L + a^2x_L^2n^2 - 2a^2x_L^2n - 2tk_1 + tax_L - ax_Ln^2t + t^2n^2 - 2t^2n + t^2)^{0.5} + 2ax_Ln - tm + t + k_1 - 2ax_L]}{-3a(n^2-2n+1)}.$$

The latter equation is the so-called response function of the follower $$x_S = f(x_L)$$. The leader will take it into account and maximize profit $$\Pi_L$$ with respect to $$x_L$$ and $$n$$:

$$\Pi_L = x_L(-a(x_L + (n-1) \cdot x_S)^2 + t(x_L + (n-1)x_S)) - k_1x_L^2.$$ 

By using the response function $$x_S$$ and differentiating with respect to $$x_L$$ and $$n$$, we get a system of two equations. The solutions of these equations are presented in Table 1.

APPENDIX 2. The expression of synergetic effect

The supply of the oligopolistic market is equal to the product quantity and the number of firms:

$$X_0 = n^* \cdot x_L^*.$$ 

By using the values from Table 1, we obtain:

$$X_0 = \frac{t}{2a}.$$ 

The supply by the monopolist is found by maximizing the profit. The profit is $$\Pi_m = P \cdot x - TC$$. Using the known expressions,

$$\Pi_m = tx^2 - ax^3 - k_1x^2 - k_2x - k_3.$$ 

The profit exceeds the maximum when $$\frac{\partial \Pi_m}{\partial x} = 0$$. From here, $$X_m$$ is equal to
\[ X_m = \frac{t - k_1 \pm \sqrt{t^2 - 2tk_1 + k_1^2 - 3ak_2 + 3ab}}{3a}. \]

The expression of the synergetic effect is \( \Delta S_x = X_0 - X_m \). By using the known expressions,

\[ \Delta S_x = \frac{t + 2k_1 \pm 2\sqrt{t^2 - 2tk_1 + k_1^2 - 3ak_2 + 3ab}}{6a}. \]