LEARNING AS A STRATEGIC PROCESS:
DEVELOPMENT OF HINTIKKA’S MODEL

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In this article learning process is studied as a strategic process. In this we have as a background information Jaakko Hintikka’s interrogative model of learning which understand all reasoning as a strategic searching process in which all the relevant factors have methodologically motivated roles. A learning process takes place in space and time: learning is an active search for new knowledge. To get a better understanding the whole framework has to be schematized. Learning as an active search is object related acting in reality. While acting an actor brings about some changes in reality. Experimentation has to be understood as methodical acting. This is a key to understand learning process as a strategic process: actions of learner and teacher have to be motivated strategically or methodologically.

Keywords: Hintikka, strategy, system transformation, high quality learning, high quality teaching, observation, and experimentation.

Professor Jaakko Hintikka has developed a general model of scientific reasoning, called interrogative model of inquiry. The basic idea behind the model is extremely simple: reasoning is understood as a process of questioning. However, the simple basic idea has shown to be extremely fruitful (Hintikka 1984, 1991, 1992; Hintikka and Bachman 1991; Sintonen 1993). The most central notion in the interrogative model for us here is the strategy. Inquiry is a strategic process. That is, methodology gives an explicit orientation to the research process. We will apply the notion of strategy to learning process in general.

The interrogative model can be seen as a general theory of reasoning (Hintikka, Halonen and Mutanen 2002). This very general nature of the model implies it is possible to apply the model in different contexts, such as scientific inquiry, argumentation, analysis of dialog, and analysis of dispute¹. Scientific inquiry is a paradigmatic example of knowledge acquisition, that is, learning. Here we mean by learning propositional learning. However, the theoretical framework we use here can also be applied to other kinds of learning. For example, in Mutanen 2005 it is applied to skill learning. The interrogative model suggests looking at learning as an interrogative process. A learning process takes place in space and time: learning is an active search for new knowledge. The nature of the newness of

¹ See Hintikka’s works in the list of references.
the knowledge plays central role separating scientific inquiry and (usual school) learning\(^2\). An aspect that characterizes learning is that usually a teacher supports it. However, the roles of learner, teacher, and additional information – both experimental and theoretical (coming from text books) – have to be specified\(^3\) (Hintikka 1982; Mutanen 2002).

To understand better, we have to schematize the whole framework. Learning as an active search is acting in reality: acting is always object related (Engeström 1999). While acting an actor brings about some changes in reality. For example, one makes a statement \(p\) to be not-\(p\). We interpret the notion of change in a broad way: one changes things also when one prevents something from happening. However, learning is not just blind acting; a learner also learns, or gets to know, some propositional knowledge whilst learning. These two aspects are interconnected to each other in several ways. These two aspects – acting (experimentation) and acquiring propositional knowledge – have to be in balance. Experiment can be seen as a tool to acquire new knowledge or evaluate knowledge one already has. The role of propositional knowledge differs in the cases.

In the paper we will develop Hintikka’s interrogative model of inquiry such that the model can be better applied in the analysis of learning process. We will relate the model to the computational epistemology developed by Kelly and Hendricks (Kelly 1996; Hendricks 2001). Moreover, to get connection to more general approaches of learning we will relate the model to the computational leaning theory (Osherson, Weinstein, and Stob 1986; Gold 1965). We will show that the Hintikka’s approach can be interconnected to the expansive learning developed by Engeström (Engeström 1999, 1999b). The more complete analysis cannot be done within this single paper.

**The notion of system**

To get a more general picture we will introduce the notions of system, system state, and system transformation (About the notions, see Kelly 1996). To specify the state of a given system we have to specify the values of given parameters (or attributes, if you like to use this notion). To keep the approach manageable we will assume that there is only a finite number of different parameters to be specified, say \(x_1, ..., x_k\). Moreover, we will assume that the values of the parameters are discrete. This discreteness is built in our very approach: the whole approach we use is discrete or rather stepwise.

To specify a state, say \(s\), of a given system \(S\) we have to determine values of the parameters \(x_1, ..., x_k\), that is, \(s = (a_1, ..., a_k)\) for some specific values \(a_1, ..., a_k\) of \(x_1, ..., x_k\) correspondingly. Let \(s\) and \(s'\) be two states. The states \(s = (a_1, ..., a_k)\) and \(s' = (a'_1, ..., a'_k)\) are identical if and only if \(a_1 = a'_1, ..., a_k = a'_k\).

We say that a system \(S\) is a – finite or infinite - set of states, that is, \(S = \{s_i : i \in I\}\), usually the index set \(I\) is the set of natural numbers. Let \(S\) be a system. At any time \(t\) there is one and only one state \(s\) that is the actual state of the system \(S\). That is, at any

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\(^2\) See Mutanen 2004; more precisely about the nature of newness, see Hendricks 2001; Hendricks and Pedersen forthcoming.

\(^3\) However, in scientific inquiry, instead of teacher, it is natural to speak of colleagues, or supervisors.
time the system is uniquely determined by the actual state. However, the notions of system and actual state only give us a static picture. Hence, to get a dynamic picture we define the following very foundational notion of system transformation relation.

Let $S$ be a system. A system transformation relation $Tr_S$ of a given system $S$ is a two place relation on $S$. The formulation $Tr_S$ denotes the foundational fact that the system transformation relation is defined relative to a given system setting. That is, the notion is a contextual and local notion. To be more accurate, contextuality denotes to the system dependence of the system transformation relation, and locality denotes to the binarity of the system transformation relation.

The domain of $Tr$ is the set $dom(Tr) \subseteq S$ such that the following holds: Let $s$ be a system state. Then $s \in dom(Tr)$ if and only if there is a $s' \in S$ such that $Tr(s,s')$. The range of $Tr$ is the set $rng(Tr) \subseteq S$ such that the following holds: Let $s$ be a system state. Then $s \in rng(Tr)$ if and only if there is a $s' \in S$ such that $Tr(s',s)$. We say that $Tr$ is complete if $dom(Tr)=S$.

The very idea of system transformation relation is to characterize the possible changes within a system that can take place. It characterizes the set of states that can be achieved from a given (actual) state $s$, that is, states that can be achieved according to system transformation relation. Iterating application of system transformation relation to a given system state gives a tree structure of system states. The root of the tree is a given (actual) state. The tree structure of system states is of central importance for us.

Let $S$ be a complete system. We say that a state $s$ in $S$ is deterministic if and only if there is one and only one $s'$ in $S$ such that $Tr(s,s')$. We say that $S$ is a deterministic system if all the states of the system are deterministic, that is, for all $s$ in $S$ there exist one and only one $s'$ in $S$ such that $Tr(s,s')$. Moreover, if for some deterministic state $s$ in $S$, $Tr(s,s)$ then $s$ would finish all the changes in the system and no changes would ever take place. In fact, the case that system transformation relation would not be complete is only a theoretical limit case, which is of no interest to us here.

A system $S$ is non-deterministic if and only if there is a $s$ in $S$ such that there is at least two states $s^*$ and $s'$ such that $s^* \neq s'$ and $(Tr(s,s^*)$ and $Tr(s,s'))$. Let us say that a system $S$ is mostly non-deterministic if most of the states, or almost all of them, are non-deterministic. That is, for most of the states $s$ in $S$, there is a set $S' \subseteq S$ such that $Tr(s,s')$ for all $s'$ in $S'$ and $Card(S')>1$. We say that the set $S'$ is the range of the relation $Tr$ in $s$. Notice that the set $S'$ depends on the state $s$. Technically such dependence is denoted by the scopes of the quantifiers (Hintikka 1995).

System transformation relation allows us to characterize the possible courses of

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4 In the following we will omit the index. However, we have to keep in mind that the relation is defined relative to a system.

5 Notice that the learner’s active role may make this implication false. We will discuss the learner’s role later.

6 Let $S$ be a given system. The completeness of $Tr$ means that for all $s \in S$ there is $s^* \in S$ such that $Tr(s,s^*)$. If $Tr$ is not complete then there is $S^* \subseteq S$ such that for all $s \in S^*$ there is $s^* \in S$ such that $Tr(s,s^*)$ and for all $s^* \in S^*$ it holds that for all $s^* \in S, Tr(s,s^*)$; moreover if $Tr$ is not complete then $S \neq \emptyset$. Then states $s^* \in S \neq \emptyset$ are halting states of the system transformation.

7 Let $S$ be a set. $Card(S)$ is the cardinality of $S$. 

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system transformations. However, this characterization is not epistemological but metaphysical (natural law). To get an epistemological or learning theoretical characterization we have to consider explicitly the actions of a learner. To determine the range $S'$ of transformation relation $Tr$ for a given system state $s$ is not an easy task: it is not easy to characterize the logical behavior of the relation $Tr$.

The parameters of the systems have to be decided case by case. In this sense systems are subjective. However, systems, after the determination of the parameters, are in a very clear sense objective structures. This does not mean, as we will see, that these structures would or should be independent of human actions or human behavior. This dependence on human actions may be of a different kind. For example, experimental set-up in physics or a work place as a system.

Our stepwise methodology means, among other things, that system transformations or chances happen, by definition, in one moment: Let $s, s' \in S$ such that at the time $t$ the state $s$ is actual state of $S$ and the next moment of time, $t+1$, the state $s'$ is actual state of $S$. In this case we say that the system state has changed. Our stepwise approach supposes that time and systems are enumerable. To generalize the approach, system changes should be considered as continuous process or systems as uncountable sets. However, we will not consider this possibility here.

Learner’s actions

In learning a learner factually makes a course of system transformations, that is, he or she chooses a branch in the tree determined by the system transformation relation. However, this choosing is not a free construction of the learner. The learner must operate within a system governed by a system transformation relation. In fact, the learner is intending to learn about the actual system and/or about the system transformation relation (Kelly 1996; Engeström 1999).

We say that a system transformation is historical if the transformation relation besides the actual state depends also on some other earlier states. In a sense, historical system transformation has a memory. In other words a system transformation is ahistorical. For example, workplace is historical in this sense. This has to be taken into account in research work. Engeström 1999 calls his historical systems activity systems. Within historical systems experimental set-ups must be planned extremely carefully keeping in mind the system is historical. Our definition above is ahistorical. We will focus on ahistorical systems.

There are different kinds of parameters in a system state. Some of the parameters are manipulable by a learner. That is, a learner may change a value of a manipulable parameter intentionally. The second class of parameters is the set of observable parameters. Of course, if a learner can manipulate a parameter he or she can also observe it. However, the converse is not true. In pure observational science the set of manipulable parameters is empty (Hintikka 1984). This

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8 Compare, for example, the accessibility relation in epistemic logic (Lenzen 1978).
indicates the sense in which pure observational science is passive: the scientist or the learner does not actively change the reality. However, he or she intentionally chooses what to observe. All the others are called theoretical parameters. They are only indirectly observable. To see the roles of different kinds of parameters let us consider the interrogative model a little closer.

The very idea is that to get to know something new a learner must acquire some additional information. This additional information must be connected to the knowledge that the learner has at the beginning of the learning process. In general this additional information comes from some outer source of information. The emphasis of the outer source is here essential. To use “inner” source means explication of the information one already has (Hintikka 1992c). These may be observations or experiments on the one hand or textbooks or inquiry reports on the other. When learning a learner intentionally searches for additional information, that is, learning is a strategic process. In such a strategic learning process the role of teacher is central. Learning in which a learner unintentionally learns is called incidental learning. However, we will not discuss it here (about it, see Mutanen 2004). To get a grasp of the activity in reality we will consider experiments more closely (Hintikka 1988; Kelly 1996).

**Experiments**

Experiments are special kinds of action in which the learner (or inquirer) intentionally changes the world. According to our terminology, in an experiment a learner intentionally changes a value of some of the parameters. That is, he or she changes the “natural” route of system transformation. He or she observes the results of the changes, and so gets further information of the system transformation relation. In an experiment knowledge about the actual system transformation is given as a result of the experiment: the values of (observable) parameters after the changes of the values of manipulable parameters. The very intention of the learner (inquirer) is to get knowledge about the system transformation relation (natural laws). To plan and make fruitful changes – ingenious experiments – is the foundational content of experimental skills.

Experiments play a central role in modern scientific thinking. Experiments act as a central tool in acquiring new additional information. However, they are not only pure questions by nature, but they also formulate the framework in which the question-answer process takes place. Hence, experimental skills are not merely the ability to make fruitful changes of the values of parameters but they are also skills to structure the whole framework of learning – of question and answer (Hintikka 1988, 1996, see also Hacking 1983 and Collins 1985).

Experimental skills presuppose some information about the object of learning, about the corresponding system. In this sense, following Kuhn, we can say that experiments are theory laden. An experimental set-up is built such that the learner focuses his or her attention on certain aspects of the system transformation; on certain dependencies on the parameters. So, again following Kuhn, we can accept
that the general framework of experiments, in a sense, determines what the learner (or inquirer) sees (See Hacking 1983).

Usually experiments take place in closed systems, that is, in systems that do not get information outside of the system. In fact, basically the very meaning of experiment presupposes a closed system. The notion of closed system is very central but also very difficult. The very methodological meaning of it cannot be observed. For example, it seems that it is paradoxical to suppose a closed system in a real world: experiments take place in a real world and hence some flow of information between a system and other parts of the reality must occur. However, here comes the real methodological meaning of experiments – these must be discussed whilst learning. For example, in chemistry washing a pipe means cutting a system transformation tree, that is, cutting down the historical development of system transformation. In other words, the learner chooses an ahistorical root of the system transformation tree. Moreover, usually the relevant branch of the tree in an experimental set-up is assumed to have only a finite number of nodes. That is, the relevant dependencies take place within a fixed finite number of steps. It is extremely important to discuss the possibility to make such cuts in different cases, and at the same teach the methodological differences and similarities of different cases. For example, in some branches of science the possibility of experiment in this sense is restricted.

The value, or fruitfulness, of the experiment is, of course, determined by the theory under learning (inquiry): how well it allows the learner to expand the theory. However, this does not mean that the experiment depends on the theory. The theory the experiment depends on may be a very different one than the theory being learning (Hintikka 1992c).

However, experimental skills, like all the other skills, depend on several different kinds of factor. We have denoted the theory dependence of experiments. Besides such theory dependence there are also more general cultural constraints. Culture entails a social pressure that causes some dispositions to the learner’s behavioral habits. These can be, for example, ethical and/or aesthetical. That is, some of the possible experimental acts are excluded for ethical reasons (experiments with human beings) or some of the hypotheses are preferred for aesthetical reasons (simplicity). However, these ethical and aesthetical factors should be justified by methodological arguments, that is, methodological constraints should explain the role of these factors (Hintikka 1984b; Mutanen 2007; 2008).

Moreover, learner’s skills – experimental and theoretical – play a central role in the learner’s orientation. In fact, in teaching one central idea is to insist on the whole experimental approach: learn to see and understand some experimental set-up in a certain way (Kuhn 1962). This creates a membership into an approach, and hence also it creates responsibility (to teacher, to university and to field of science) to the learner.

Here we have a kind of constructivist mode. However, as we will see, this does

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9 Note that this is a stronger condition than the mere ahistoricity of the system.
not mean that a learner could construct anything he or she likes to construct nor does this mean that a learner would construct his or her knowledge during learning. Here our emphasis contradicts the idea of (social) constructivism (Burr 1996). Our basic attitude here is realistic. To get a better grasp, let us consider more closely the relationship between experimental and theoretical aspects in learning.

**Interrogation and experimentation**

In Hintikka’s interrogative model the very foundational idea is that the learner uses the additional information coming from some outer source of information actively during learning. The learner wants to get to know something, say \( C \). He or she is trying to infer \( C \) from the knowledge at the beginning of the learning process \( K \) – called *background knowledge* – and the additional information \( D \):

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K \cup D \vdash C.
\]

In fact, this formulation indicates the fact that theoretical and experimental information has to interact with each other\(^\text{10}\).

Here it must be emphasized that both background knowledge and additional information is necessary in learning. In fact, there is formal proof that the actual learning process is not possible without theoretical (non-observational) parameters (Gaifman, Osherson and Weinstein 1990).

Additional information contains both observational and experimental information. That means, for example, that new individuals will be introduced (experimental parameters). The central methodological role these new individuals have is to give a richer picture about the reality by considering individuals together in their relation to each other. In education it is extremely important to emphasize this aspect. However, mostly this is not observable; it must be explicated *discursively*.

However, the information conveyed by the experimental parameters is also used explicitly in forming propositional knowledge (Hintikka 1984; Mutanen 1997). However, the content of the set \( D \) must be specified case by case. So, the methodological role of the set \( D \) must be emphasized over and over again.

The fact that learning is a process must be taken into the formalism. This can be done by using *functional* stepwise analysis. That is, a learner will be identified with a learning function. The identification of the learner with a function may seem a little strange. However, our methodological approach justifies the identification: from the methodological point of view the central problem is whether or not a *method* learns the thing to be learned: a learner uses a method. From the methodological point of view there is no reason to separate a method and a learner (Hendricks 2001; Mutanen 2004).

A learner, as we have seen, is a skilful actor who has, besides experimental skills, some propositional knowledge or *background knowledge*. These – experimental skills and background knowledge – describe the general framework in which the learner learns, that is, these determine, partly, the *range of attention* of the learner, that is, the

branches determined by the system transformation relation that the learner considers possible ones. Based on background knowledge and experimental skills, the learner characterizes, more or less explicitly, an hypothesis the learner intends to learn and experimental set-ups that he or she intends to do. The arguments must present these two kinds of parameter. Let us represent these by the vectors $h$ and $e$. So, a learning function is of the form $F(h,e)$.

The arguments of the skill functions determine the epistemic goal and the experience the learner has. In general, the goal need not be, and usually is not, explicitly expressed. The epistemic goal is not yet grasped very well especially, at the beginning. The function explicates the learning strategy of the learner: it tells what to do in any given situation. The class of values of the function is partly determined by the background knowledge which the learner has at the beginning of the learning process (Kelly 1996; and Mutanen 2004).

The values of the skill function characterize the present situation, the experimental acts to be done and the parameters that are under the learner’s control. In fact, the degree of explicitness of the characterization characterizes the theoretical progress in learning. The second sequence of parameters of the value characterizes the experimental progress in learning. Usually these two are emphasized both by learners and teachers (evaluators). However, as we have noted, there are also methodological aspects that are not directly observable – what possible knowledge. Let the set $V$ of parameters denote to the set of parameters that are under the learner’s control; parameters that he or she can – intentionally – vary. So, experimental skills are defined as skill functions: $F(h,e) = (c,m,V)$.

However, strategically most important is the third sequence, namely, the set of parameters that are under the learner’s control. This indicates the degree of freedom that the learner has. To learn intentionally and to get a good grasp – high quality learning - the learner has to learn the interdependence of different kinds of parameter. In high quality learning, the key idea is the strategic use of the background knowledge and experimental skills the learner has.

**Teaching**

In traditional school teaching – this is also the case sometimes in university teaching - the teacher just tells the learner facts – both theoretical and experimental (what one must know and do). In traditional university teaching a learner (student) just follows (or imitates) a professor – master-journeyman-model. That is, two similar functions have parallel “runs”. However, these traditional teaching models can be characterized as closed strategy models of teaching. That is, teaching just gives arguments to the learner’s strategy function. However, in such teaching it is extremely difficult to learn a strategy (Gold 1965).

In strategic, or methodological, teaching, the high quality teaching model, the teacher underlines the learner’s strategy functions. The teacher allows the learner to explicate and change strategy functions. Interestingly enough, this strategic, high quality, model of teaching shows a formal character of teaching, and hence could be computer simulated.
To understand more closely the idea of high quality teaching we must, once again, emphasize that learning is a strategic process. In high quality teaching the emphasis is focused on the methodological use of background knowledge and experimental skills. That is, the emphasis is on what possible questions. This means that instead of teaching the factual experimental set-ups or explicit formulation of the underlying theory, the teaching will focus on the role of theory and experiments in learning. The idea is to emphasize the methodical role of background knowledge and experiment, that is, the role of theory and experiments in learning. The idea is to emphasize the methodical role of background knowledge and experiment, that is, the role of theory and experiments in learning.

To make the idea clearer we must consider a little the role of the set $V$. The set denotes to the parameters under the learner’s control. However, this set explicates the (experimental) possibilities that the learner has. So, in high quality teaching and learning understanding the nature and role of the set $V$ plays a central role. Here we have a central difference between high quality teaching and learning: traditionally the emphasis in teaching and learning has been on the first two argument sequences of the value sequence, that is, on the explicit propositional knowledge and experimental settings. However, in evaluating the knowledge one needs something more – something that could be known or done. Sometimes denoting the central role of mistakes in learning emphasizes this (for example, see Turing 1950). Here we have to emphasize that mistakes play a central role in every kind of teaching and learning but in high quality teaching the strategic role of the mistakes is emphasized: mistakes open new lines of thoughts.

We can divide mistakes into two classes: factual mistakes and methodological mistakes. The first class includes both incorrect propositional knowledge and experimental mistakes. That is a learner may have false opinions or he or she may conduct experiments incorrectly. These are in a sense closed mistakes: opinions are either true or false and experiments are done correctly or incorrectly. These are local mistakes in the sense explained above.

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11 The notion of direct knowledge denotes knowledge that is in a sense non-reflective – or non-modal as we could also say.

12 Here we use a dichotomy realized in actual teaching – however, everyone who has taught at any educational level can recognize these different aspects in practice.
However, methodological mistakes are strategically wrong steps in learning. This means that evaluation of the steps must be done reflecting the whole learning process. That is, the same step may be correct or wrong depending on the learning in which it occurs. So, these are contextual mistakes in the sense explained above. Hence, also evaluation must be contextual (Engeström 1999). Moreover, evaluation must be done by cooperating together with a learner.

In experimentation (both in research and learning) it is central to have a strategic plan. However, this plan is usually implicit. That is, in research reports and explicit research plans there is no explicit characterization of this. All first grade inquirers are acquainted with this strategic aspect of inquiry. The very idea in high quality teaching is to help the learner through discussion explicate a methodological framework in which he or she can successfully learn.

In the philosophy of science this aspect has been under extensive study in recent decades under the topic ‘Logic of discovery’. One basic idea has been that the context of discovery must be explicated. That is, discoveries take place only within a context. So, discovery is a contextual notion (in a sense characterized above). A natural way to find new interconnections between parameters is to introduce new individuals (parameters) into consideration (Hintikka 1973).

REFERENCES


**Arto Mutanen**

**SANTRAUKA**


**Pagrindiniai žodžiai:** Hintikka, strategija, sistemos pokytis, aukstos kokybės mokymasis, stebėjimas, eksperimentavimas.