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INVESTMENT PORTFOLIO'S INCLUDING DIFFERENT CRYPTOCURRENCIES EFFICIENCY STUDY

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ABSTRACT. *The development of technology and globalisation have opened wide opportunities even for investors without relevant knowledge or experience to use investment instruments. Although cryptocurrencies have unique features that attract investors, the market is also subject to risks: price fluctuations, operational risks, and the issue of trust. In this context, looking for effective solutions for investment portfolios and being able to understand better and evaluate the cryptocurrency market becomes especially important. Previous studies have highlighted the benefits of cryptocurrencies, such as diversification, hedging, and safe havens, but there is little research that reveals the performance of a cryptocurrency portfolio. Investors seek the highest possible return but generally avoid or underestimate risk. Applying portfolio optimisation in the study aims to identify cryptocurrencies, the inclusion of which in a traditional asset investment portfolio would help to achieve the most effective combination of desired return and risk. The primary purpose of this study is to evaluate the impact of different cryptocurrencies on the efficiency of the investment portfolio. Ten cryptocurrencies included in the B10 BITA CRYPTO 10 INDEX composition are studied. Due to the instability of cryptocurrencies, the composition of this index is constantly changing. The cryptocurrencies analysed in the study, included in the composition of this index, were fixed in August 2023. Traditional assets in the study are represented by stock indices: S&P 500, Euro Stoxx 50, DAX and CAC 40. After identifying efficient portfolios in the study, it can be concluded that the inclusion of different cryptocurrencies in an investment portfolio consisting of stock indices does not always allow efficient portfolio results to be achieved.*

KEYWORDS: cryptocurrency, diversification, efficient portfolio, portfolio optimisation, stock indices, traditional asset.

JEL classification: G11.

Introduction

The cryptocurrency market is an emerging asset class that has attracted significant and growing attention from investors globally (Tripathi *et al.*, 2022). Investment in cryptocurrency-related assets is considered by many funds and portfolio managers (Bouri *et al.*, 2022). Bouri *et al.* (2022) emphasise that cryptocurrencies remain an attractive area of research that has atypical characteristics. Kumar *et al.* (2022) argue that the cryptocurrency market is an interesting and ever-evolving system that deserves further investigation. Hasan *et al.* (2022) state that since 2018 reputable financial institutions have participated in the cryptocurrency market. The increasing population interest in cryptocurrency is mainly due to the opportunity to invest money and get superprofits due to the growth of cryptocurrency quotations (Zaiets, Yeskov, 2021). The main criteria for investing in cryptocurrencies are the ratio of potential risk and potential benefits investors can get from ownership (Mikhaylov, 2020).

Hsu *et al.* (2021) argue that previous studies have classified cryptocurrency's capabilities into diversification, hedging or safe haven. Cryptocurrencies have recently become an alternative class of digital assets, but little is known about their portfolio

performance (Demiralay, Bayracı, 2021). Operating with new digital assets is important for improving companies' digital transformation and literacy (Krajčík *et al.*, 2023). The efficiency of the portfolio with the inclusion of cryptocurrencies was studied by Andrianto and Diputra (2017), Inci and Lagasse (2019), Li *et al.* (2021). Andrianto and Diputra (2017), using modern portfolio theory as a quite effective basis for establishing portfolios, found that cryptocurrency increases the effectiveness of the portfolio. Inci and Lagasse (2019) highlighted that cryptocurrencies are useful in optimal portfolio building and investment. Moreover, they argue that the growing popularity and unique characteristics of cryptocurrencies will help them become a future investment portfolio component.

The fourth industrial revolution has brought the growing use of technology in all areas of life, including finance and investment (Li *et al.*, 2021; Khalatur *et al.*, 2022). This industrial revolution laid the foundations for blockchain technology and opened the way for cryptocurrencies (Demiralay *et al.*, 2021; Korauš, *et al.*, 2021). The development of technology and globalisation has opened wide opportunities even for investors without relevant knowledge or experience to use investment instruments. Transaction volumes in crypto assets on the chain have increased rapidly, especially during the COVID-19 pandemic (Feyen *et al.*, 2022; Stoyanova *et al.*, 2022; Lincényi, Mindár, 2022; Czechowska, Florczak, 2022). Feyen *et al.* (2022) argue that cryptocurrency activity seems to be a global phenomenon. The ability to properly manage risk and achieve higher returns by including cryptocurrencies in a portfolio would open up more investment opportunities. Conversely, empirical studies highlight a pronounced correlation between strict compliance with the established rules and proficient risk management practices (Cernisevs, *et al.*, 2023; Radavičius, Tvaronavičienė, 2022).

In a previously conducted study by the authors Juškaitė and Gudelytė-Žilinskiėnė (2022), the aim was to identify cryptocurrencies suitable for investment and portfolio diversification. This study aims to determine the impact of the inclusion of different cryptocurrencies in an investment portfolio consisting of traditional assets on its efficiency. The study aims to include all the cryptocurrencies that make up the B10 BITA CRYPTO 10 INDEX in the investment portfolio.

The problem of the research. Can the inclusion of different cryptocurrencies in an investment portfolio affect its efficiency?

Research object. Including cryptocurrency in the investment portfolio.

The article aims to evaluate the influence of different cryptocurrencies on the efficiency of the investment portfolio.

Tasks of the research:

1. To analyse the theoretical aspects of the influence of cryptocurrencies on the efficiency of the investment portfolio.
2. To make the study methodology for the efficiency of the investment portfolio, including different cryptocurrencies.
3. To study the efficiency of the investment portfolio, including different cryptocurrencies.

Research methods. Scientific literature analysis, bibliometric analysis, statistical data analysis, portfolio optimisation.

Research limitations. Ten cryptocurrencies included in the composition of the B10 BITA CRYPTO 10 INDEX are studied. Due to the instability of cryptocurrencies, the composition of this index is constantly changing. The cryptocurrencies analysed in the study,

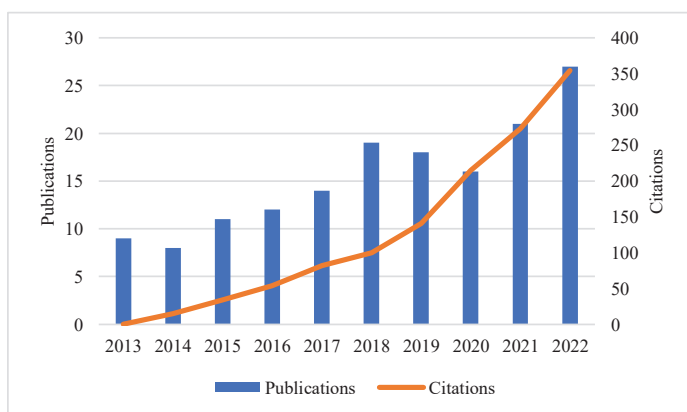
included in the composition of this index, were fixed in August 2023. Traditional assets in the study are represented by stock indices: S&P 500, Euro Stoxx 50, DAX and CAC 40.

1. Theoretical Analysis of the Influence of Cryptocurrencies on the Efficiency of the Investment Portfolio

Recently, an increasing number of cryptocurrency funds have been added to the category of crypto assets and, therefore, into the investment universe (Brauneis & Mestel, 2019). The scientific literature raises the question of whether cryptocurrencies could be one of the alternatives to improve portfolio risk management. Portfolio theory supports the diversification of investments by allocating assets strategically to maximise returns for a given level of risk (Fang *et al.*, 2022). Chen *et al.* (2022) identify *asset allocation* as an important investment strategy that balances reward and risk by dividing portfolio assets based on individual goals, risk tolerance, and investment horizon. Portfolio construction is the basis of investment management processes (Mba *et al.*, 2022).

The basis of portfolio allocation over the past half century has been Markowitz mean-variance portfolio selection, which remains the most essential tool in the portfolio literature. (Huang *et al.*, 2022). Chen *et al.* (2022) defined the published Markowitz portfolio selection model in 1952 as theoretically very successful.

From the first figure, it can be seen that the growing relevance of the Markowitz portfolio theory is observed when analysing the sources of scientific literature in the Web of Science database. After setting the keywords “Markowitz portfolio” and performing a search in the Web of Science database, 155 published publications were obtained in 2013–2022. The number of citations also increased consistently during this period. Based on the graph presented in *Figure 1*, it can be seen that the amount of research related to Markowitz portfolio theory and the number of citations tend to increase.



Source: compiled by the authors based on Web of Science, n.d. database.

Figure 1. Times Cited and Publications by Keywords “Markowitz Portfolio”

Sadeghi *et al.* (2022) argued that Markowitz and his groundbreaking research in 1952 developed a revolutionary portfolio risk return model that since then has been widely used to determine the optimal portfolio. After Harry Markowitz published an innovative paper on portfolio selection in 1952, A. D. Roy also published a paper about three months later arguing

for portfolio selection using the mean-variance criterion (Sullivan, 2011). Essentially, these economists developed the same theory of portfolio selection at the same time, independently of each other (Sullivan, 2011). In Markowitz's portfolio theory, the following terms were distinguished:

- Portfolio risk;
- Portfolio diversification.

Markowitz (1999) claimed that diversification of investments was a practice that existed long before he published his paper on portfolio selection. However, before 1952, there was no adequate investment theory covering the effects of diversification when risks are correlated, distinguished between efficient and inefficient portfolios, and analysed the risk return balance in the portfolio as a whole. Arguably, one of the most important strengths of Markowitz's portfolio theory is its ability to identify the overall risk of a portfolio, which can be managed through the application of diversification.

Efficient portfolios have increasing risk and returns, which is acceptable to investors, while each investor decides his or her acceptable level of risk (Živkov *et al.*, 2022). Inefficient portfolios have an increasing risk with reducing returns, which is not a suitable choice for each investor (Živkov *et al.*, 2022). In the expected average return, the problem of selecting a standard deviation portfolio is usually the first step to obtaining a set of efficient portfolios, represented by the efficient frontier in the space of objective function values (Juszczuk *et al.*, 2022). By creating an efficient frontier, the best combinations of investments that offer the highest return rate for a particular risk rate within an investment portfolio can be observed (Ćosić, Čeh Časni, 2019). The efficient frontier is the basis of modern portfolio theory (Liu, 2022). It helps investors analyse possible risks and returns and then choose the best investment strategy (Liu, 2022). The efficient frontier can be defined as a set of different portfolio scenarios where maximum returns are achieved at minimal risk under return-risk constraints (Guo, 2022).

With the development of the global economy, science and technology, and the development of the theory of stock portfolios, sophisticated investors consider return and risk management to be their priority (Sun, 2022). Markowitz's model focuses on maximising returns and minimising risks at the same time (Logubayom, Victor, 2019). It should be able to achieve a single portfolio of risk assets with the lowest possible risk than any other portfolio with the same level of return (Logubayom, Victor, 2019). Markowitz's model uses variance to represent portfolio risk and mean to represent expected portfolio return (Guo, 2022).

Markowitz's portfolio theory proposed a model that allowed investors to construct an optimal portfolio by selecting the optimal combination of expected return and risk. This model showed investors the benefits of diversification. In addition, the optimisation process produces efficient portfolio frontiers that can all be considered optimal regarding the risk-return ratio since the return is maximised for any given risk level (Schellinger, 2020).

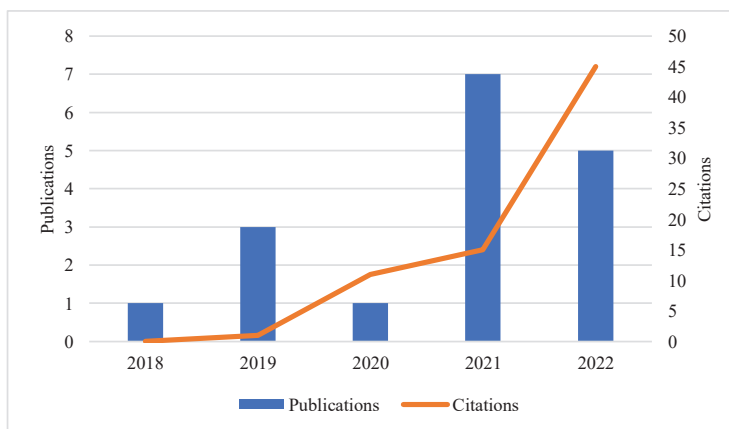
The inclusion of various asset classes has an impact on an investment portfolio and improves or reduces its performance. (Letho *et al.*, 2022). Alternative investments to improve the returns of traditional stock and bond portfolios have long been the subject of academic research, but the results are mainly mixed and sometimes contradictory (Matkovskyy *et al.*, 2021). When the sources of scientific literature in the Web of Science database are analysed, the continuing relevance of investment portfolio efficiency research is observed. After selecting the keywords "portfolio", "efficiency" and "investment" and performing a search in the Web of Science database, 965 published publications were obtained in 2013–2022. These

portfolio efficiency. According to Schellinger (2020), adding various cryptocurrencies to the portfolio could have an acceptable overall risk, while optimal selection of individual weights in the portfolio can further improve performance.

Analysing sources from the 2018–2022 scientific literature related to the inclusion of cryptocurrencies in the investment portfolio in the “Web of Science” database using the VOSviewer software, a keyword network visualisation map was compiled. A network visualisation of the most used keywords is presented in *Figure 3*. When analysing the data in *Figure 3*, it can be seen that the keywords “cryptocurrency” or “cryptocurrencies” are related to keywords “efficiency”, “inefficiency”, “diversification”, “portfolio optimisation” and “portfolio diversification”. The network nodes of the keywords “efficiency”, “inefficiency”, “diversification”, “portfolio optimisation” and “portfolio diversification” are smaller compared to the network nodes of the keyword’s “cryptocurrency”, “cryptocurrencies” or “bitcoin”. Based on this, it can be argued that there is still little research on the inclusion of cryptocurrencies in a portfolio and its efficiency.

Trimborn *et al.* (2020) reveal that including cryptocurrencies in a portfolio increases returns compared to portfolios consisting only of traditional assets and can improve the risk-return ratio. Although cryptocurrencies have extreme volatility, their high average returns and low correlation have made them an alternative investment asset for portfolio and risk management (Petukhina *et al.*, 2021). According to the results of a study by Andrianto and Diputra (2017), a portfolio with cryptocurrency increases portfolio efficiency by creating more allocation options for investors.

Letho *et al.* (2022) found that including cryptocurrencies in a portfolio increases the financial gain of the portfolio and provides diversification benefits. Matkovskyy *et al.* (2021) note that on purpose to increase returns, including cryptocurrencies, add value to traditional stock portfolios. The results of the Bondar *et al.* (2020) study show that investment portfolio efficiency increases due to the inclusion of cryptocurrencies and risk can be controlled despite the high volatility of cryptocurrencies. The work by Aliu *et al.* (2022) shows that stocks are the most appropriate asset to improve risk in cryptocurrency portfolios.



Source: compiled by the authors based on Web of Science, n.d. database.

Figure 4. Times Cited and Publications by Keywords “Portfolio”, “Efficiency”, “Investment” and „Cryptocurrency”

Analysing the sources of scientific literature in 2018–2022 in the “Web of Science” database under the keywords “portfolio”, “efficiency”, “investment” and “cryptocurrenc*”, an increase in the number of publications in 2021–2022 is observed. These data are presented in the diagram in *Figure 4*. The diagram presented in *Figure 4* shows the novelty and the growing relevance of the topic of effectiveness of the investment portfolio of research involving cryptocurrencies. An apparent increase in the number of research citations on this topic was also observed.

In the literature, special attention is paid to the pioneer of cryptocurrencies – Bitcoin. This trend can also be seen in the visualisation network in *Figure 3*. The keyword “bitcoin” network node is one of the largest. This cryptocurrency is the market leader and influences the price changes and trading volumes of other digital currencies (Aliu *et al.*, 2022). Tomić (2020) notes that Bitcoin is the first digital currency that relies on asymmetric cryptography; therefore, its value changes can be a key indicator of the movement of the entire cryptocurrency market.

Kumaran (2022) writes that adding Bitcoin to a portfolio significantly improves its performance, mainly due to higher returns rather than decreased risk. Boiko *et al.* (2021) formed an optimal portfolio of cryptocurrencies using a modified Markowitz model, the results of which showed that the dominance of Bitcoin in the cryptocurrency portfolio is due to its high profitability and low risk. Bakry *et al.* (2021) investigate the potential of Bitcoin as a diversifier under different portfolio optimisation strategies. Chou Awara ir Tchitchoua (2019) studied the contribution of Bitcoin in improving investors’ portfolios and found that portfolios with Bitcoin have a better risk-reward ratio compared to portfolios without Bitcoin. Som and Kayal (2022), analysing the price dynamics of the four largest cryptocurrencies by market capitalisation and various portfolios, argue that Bitcoin should be included in the portfolio, perhaps as a small percentage, because it significantly increases returns with the risk acceptable to the investor.

The diversification property of cryptocurrencies is emphasised in the scientific literature. Boiko *et al.* (2021) emphasise that diversification is an important tool for managing risk, a practical example of which is the creation of a portfolio of financial assets. Cryptocurrencies are relevant for the diversification benefits they provide in an investment portfolio, but Aliu *et al.* (2022) note that research on the diversification benefits of cryptocurrencies is still limited. Huang *et al.* (2022) argue that the less risk-averse investors are, the more valuable cryptocurrencies are for portfolio diversification.

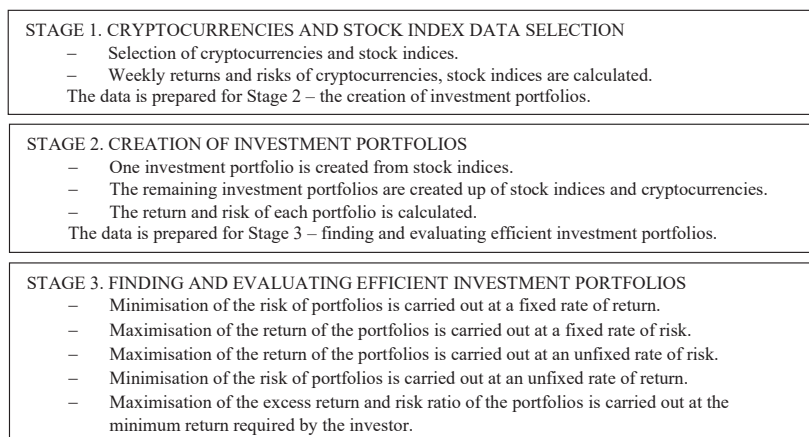
Trichilli and Abbes (2023) write that the possibility of using cryptocurrency to insure traditional assets has been of interest to investors, professionals and academics since the beginning of cryptocurrencies. However, Petukhina *et al.* (2021) note that the role of cryptocurrency in investors’ portfolios is not entirely clear and remains controversial. When including cryptocurrencies in the portfolio, it is necessary to pay attention to their low liquidity compared to traditional assets (Trimborn *et al.*, 2020). If one wants to include cryptocurrencies and stocks in the same portfolio, one should avoid giving too much weight to cryptocurrencies, as this can cause liquidity problems when reallocating the portfolio (Trimborn *et al.*, 2020). Kumaran (2022) points out that real-time investment decisions must be verified based on market scenarios.

Based on the analysed scientific literature and conducted research, it can be stated that the inclusion of cryptocurrencies in the investment portfolio of other assets could positively affect its efficiency. The low correlation observed between cryptocurrencies and other assets and the possible high returns could serve as a counterweight to the high risk of

cryptocurrencies. By including cryptocurrencies in the investment portfolio of other assets, it is possible to achieve a lower overall portfolio risk or increase the overall expected return of the portfolio and discover more efficient asset allocation and investment opportunities. Investors care about both return and risk, so H. Markowitz proposed a portfolio theory that helps an investor choose an optimal portfolio to ensure maximum return at a given level of risk and minimum risk at a given level of return. The theory of portfolio selection presented by A.D. Roy, which uses the mean-variance criterion, is essentially very similar to Markowitz. Markowitz's portfolio theory proposed a model that enabled investors to construct an optimal portfolio by selecting the optimal combination of expected return and risk. More attention is paid to research on cryptocurrency – Bitcoin. It is not surprising since this cryptocurrency remains the leader of the cryptocurrency market in terms of capitalisation. At the same time, it can be observed that there is a growing number of studies investigating other cryptocurrencies and the possibility of including a larger number of them in the investment portfolio.

2. Research Methodology

The purpose of the research is to evaluate the impact of different cryptocurrencies on the efficiency of the investment portfolio. This research consists of three stages. The research scheme is presented in *Figure 5*. Excel software is used for research.



Source: compiled by the authors.

Figure 5. Research Scheme

This research on the efficiency of investment portfolios aims to identify cryptocurrencies that could be used as an alternative and attractive investment instrument to improve the portfolio's return and risk ratio.

In Stage 1 of the research, cryptocurrencies that make up the B10 BITA CRYPTO 10 INDEX are selected, and data are available in the research period from 01-08-2022 to 01-08-2023. Traditional assets in research are reflected in stock indices, which are part of traditional assets in investment portfolios. Weekly returns are calculated for selected cryptocurrencies and stock indices. For data compatibility, the weekly returns of investment instruments are

chosen for this research since cryptocurrencies are traded daily, and stocks are only on working days. Cryptocurrency and stock index data sourced from Yahoo Finance, n.d.

Based on Inci and Lagasse (2019), the weekly returns of cryptocurrencies and stock indexes are calculated by the following formula:

$$R_t = \ln(P_t) - \ln(P_{t-1}), \quad (1)$$

where: R_t – weekly return on the investment instrument; P_t – weekly closing price of the investment instrument; P_{t-1} – closing price of the investment instrument of the previous week.

The risk of the investment instruments analysed in the research is assessed by calculating the standard deviation. The standard deviation is calculated using the STDEV.P function in Microsoft Excel (*STDEV.P Function*, n.d.). Using this Excel function, the risk of the investment instrument for the period under investigation is calculated according to the following formula:

$$\sigma = \sqrt{\frac{\sum(R - \bar{R})^2}{n}}, \quad (2)$$

where: σ – standard deviation of the weekly return of the investment instrument (risk); R – weekly return of the investment instrument; \bar{R} – average weekly returns of the investment instrument; n – number of observations.

In the second stage of the research, the first portfolio is formed, which consists only of traditional assets – stock indices. The rest of the investment portfolios are formed from the same stock indices and cryptocurrencies selected for the research. At this stage, the investment instruments in the portfolios are distributed equally. Once the portfolios are formed, their returns and risks are calculated.

The expected return of the investment portfolio is equal to the average of the expected returns of the instruments included in the portfolio composition (Žilinskij, 2012):

$$E(R_p) = \sum_{i=1}^n \omega_i E(R_i), \quad (3)$$

where: $E(R_p)$ – expected portfolio return; ω_i – the weight of the investment instrument in the portfolio; $E(R_i)$ – i expected return of the investment instrument; n – number of investment instruments in the portfolio.

The risk of an investment portfolio is measured using the standard deviation of the expected return of the portfolio (Žilinskij, 2012):

$$\sigma_p = \left[\sum_{i=1}^N \sum_{j=1}^M \omega_i \omega_j \text{COV}(R_i R_j) \right]^{1/2}, \quad (4)$$

where: σ_p – portfolio risk; ω_i, ω_j – i and j investment instrument weights in the portfolio; $\text{COV}(R_i R_j)$ – covariance between investment instruments i and j .

The covariance of returns on investment instruments is calculated using the COVARIANCE.P function in the Microsoft Excel analysis package (*COVARIANCE.P Function*, n.d.). Based on this function, the covariance is calculated according to the following formula:

$$COV(R_i R_j) = \frac{\sum (R_i - \bar{R}_i)(R_j - \bar{R}_j)}{n}, \quad (5)$$

where: $COV(R_i R_j)$ – covariance between investment instruments i and j ; R_i, R_j – return of investment instruments i and j ; \bar{R}_i, \bar{R}_j – average returns of investment instruments i and j ; n – number of observations.

In Stage 3 of the research, they are optimised to evaluate the efficiency of the investment portfolios. Optimising an investment portfolio in order to find the most acceptable ratio of return and risk is based on portfolio theory. According to Markowitz's portfolio theory, there is a limit to the efficient portfolio that allows minimising risk at a given level of profitability or maximising profit at a given level of risk (Žilinskij, 2012). Portfolio optimisation can be formulated in two ways: $\max E(R_p)$ and $\min \sigma^2$ (Paškevičius, Kupreščenkaitė, 2011). Mechanically calculating the optimal weight of each asset in a portfolio is difficult, so Microsoft Excel SOLVER is used, a tool for determining the optimal set of asset weights in a portfolio (Paškevičius, Kupreščenkaitė, 2011). For this research, the following mentioned Microsoft Excel Solver function is used to determine the optimal portfolio.

Minimising portfolio risk with fixed returns:

$$\begin{cases} \min (\sigma_p) \\ E(R_p) = C \\ \sum_{i=0}^n \omega_i = 1 \\ \omega_i \geq 0 \end{cases}, \quad (6)$$

where: σ_p – portfolio risk; $E(R_p)$ – expected portfolio return; ω_i – the weight of the investment instrument in the portfolio.

Maximising portfolio returns with fixed risk:

$$\begin{cases} \max E(R_p) \\ \sigma_p = C \\ \sum_{i=0}^n \omega_i = 1 \\ \omega_i \geq 0 \end{cases}, \quad (7)$$

where: $E(R_p)$ – expected portfolio return; σ_p – portfolio risk; ω_i – the weight of the investment instrument in the portfolio.

Further, the research looks at what the maximum return and minimum risk of investment portfolios would be and how the weights of the investment instruments would be distributed in the absence of a fixed return or risk.

Maximising portfolio returns with unfixed risk:

$$\begin{cases} \max E(R_p) \\ \sum_{i=0}^n \omega_i = 1 \\ \omega_i \geq 0 \end{cases}, \quad (8)$$

where: $E(R_p)$ – expected portfolio return; ω_i – the weight of the investment instrument in the portfolio.

Minimising portfolio risk with unfixed returns:

$$\begin{cases} \min (\sigma_p) \\ \sum_{i=0}^n \omega_i = 1 \\ \omega_i \geq 0 \end{cases}, \quad (9)$$

where: σ_p – portfolio risk; ω_i – the weight of the investment instrument in the portfolio.

The literature analysis shows that the Sharpe ratio is often used to evaluate the efficiency of the investment portfolio. A higher Sharpe ratio value is better because it indicates that there is a better reward for the standard deviation (risk); in other words, the portfolio will be more efficient (Paramitha, Faturohman, 2022).

Sharpe's theory is very similar to previously published Roy's theory. The main difference between Roy and Sharpe is that Roy proposes to use the investor's minimum returns instead of risk-free interest rates (Žilinskij, 2012). When presenting his model, Roy states that the expected returns of investment (m) should not be lower than a certain level of negative consequences (d) (Sullivan, 2011). Roy's statement can be restated in this formula (Sullivan, 2011):

$$\text{Maximising } \frac{m-d}{\sigma}, \quad (10)$$

where: m – expected return on investment; d – minimum required return; σ – standard deviation of returns.

Based on this, finding the efficiency of the investment portfolio in this research is done by maximising the ratio of excess return to risk:

$$\begin{cases} \max \left(\frac{E(R_p) - d}{\sigma_p} \right) \\ \sum_{i=0}^n \omega_i = 1 \\ \omega_i \geq 0 \end{cases}, \quad (11)$$

where: $E(R_p)$ – expected portfolio return; d – minimum return required by an investor; σ_p – portfolio risk; ω_i – the weight of the investment instrument in the portfolio.

Sullivan (2011) states that Markowitz's proposed diversification reduces portfolio return variance, while Roy's diversification is a tool to reduce the probability of negative consequences.

After calculating the returns and risks of the portfolios and applying portfolio optimisation based on the Markowitz and Roy theories, it is possible to evaluate how the inclusion of cryptocurrencies can affect the efficiency of the investment portfolio.

3. Research Results

In the first stage of the research, the cryptocurrencies that make up the B10 BITA CRYPTO 10 INDEX are selected. The research period is from 1 August 2022 to 1 August 2023. The cryptocurrency index on 15 August 2023 included: Bitcoin BTC, Ethereum ETH, Binance Coin BNB, Dogecoin DOGE, Cardano ADA, Solana SOL, TRON TRX, Polkadot DOT, Shiba Inu SHIB, Avalanche AVAX (BITA | *The Index Technology Company*, n.d.). All of these cryptocurrencies were traded during the research period, so the data for all cryptocurrencies are used in the investment portfolio efficiency research. In the research, stock indices reflect the share of traditional assets: S&P 500 (American stock market), Euro

Stoxx 50 (European stock market), DAX (German stock market), CAC 40 (French stock market). The weekly closing prices of the cryptocurrencies and stock indices studied are obtained from Yahoo Finance, n.d.

In order to assess the efficiency of the investment portfolio when including cryptocurrencies, the weekly returns of the selected investment instruments are calculated according to formula (1) and the standard deviations (risks) according to formula (2) presented in the methodology. The data obtained are presented in *Table 1*.

Table 1. Weekly returns and standard deviations

Investment instrument	Average weekly return	Standard Deviation / Risk
Bitcoin BTC	0.48 %	7.44 %
Ethereum ETH	0.19 %	8.84 %
Binance Coin BNB	-0.51 %	7.45 %
Dogecoin DOGE	0.24 %	13.12 %
Cardano ADA	-1.02 %	9.64 %
Solana SOL	-1.01 %	18.38 %
TRON TRX	0.23 %	5.25 %
Polkadot DOT	-0.98 %	8.65 %
Shiba Inu SHIB	-0.78 %	10.93 %
Avalanche AVAX	-1.49 %	11.45 %
S&P 500	0.20 %	2.44 %
Euro Stoxx 50	0.32 %	2.33 %
DAX	0.34 %	2.26 %
CAC 40	0.26 %	2.30 %

Source: compiled by the authors.

From *Table 1*, it can be seen that Bitcoin had the highest average positive return over the research period. During this period, it was observed that all the stock indices studied had a positive average weekly return, while some of the cryptocurrencies studied had a negative average weekly return. In this period, the studied stock indices had not only positive returns but also the lowest risk, while some cryptocurrencies had not only negative returns but also higher risk compared to stock indices.

In the second stage of the research, analysed investment portfolios are created, including selected cryptocurrencies, to evaluate their influence on the efficiency of the traditional asset portfolio. The expected returns of these portfolios are calculated based on the historical data of the investment instruments and assuming that the instruments will maintain the specified returns for a certain number of periods. Prior to portfolio optimisation, investment portfolios are created in equal parts. According to formulas (3) and (4) presented in the research methodology, the average weekly returns and risks of the investment portfolios formed are calculated. The first portfolio consists only of traditional assets: all selected stock indices. Other research investment portfolios include cryptocurrencies. When observing in the literature the special attention paid to the pioneer of the cryptocurrency Bitcoin, it was decided to make a second portfolio from all the selected stock indices and the cryptocurrency Bitcoin. According to the results of the ranking and correlation analysis of cryptocurrencies published by Juškaitė and Gudelytė-Žilinskiėnė (2022), the third investment portfolio is created of all investigated stock indices, including cryptocurrencies Bitcoin, Ethereum and Dogecoin. The fourth portfolio consists of all selected stock indices and all ten cryptocurrencies. The investment portfolios consisted of equal parts, and the calculation results were based on the weekly returns of the investment instruments, including different cryptocurrencies, which are

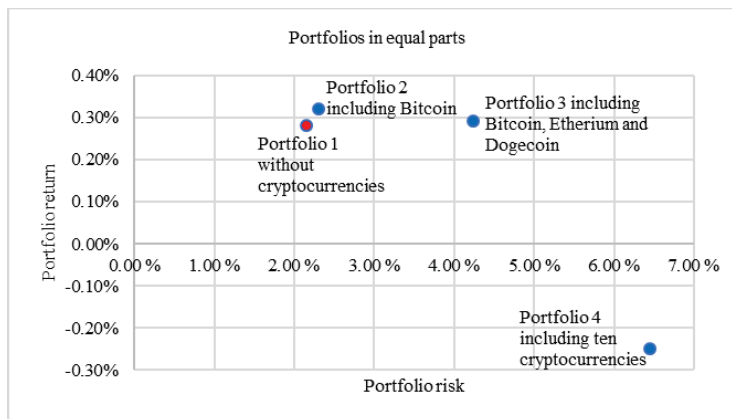
presented in *Table 2*. The distribution of all investment portfolios based on the calculated portfolio returns and risks is presented in *Figure 6*.

Table 2. Portfolios in equal parts

Investment portfolio	Average return	Standard deviation / Risk
Portfolio 1 (without cryptocurrencies)	0.28 %	2.16 %
Portfolio 2 (including Bitcoin)	0.32 %	2.31 %
Portfolio 3 (including Bitcoin, Ethereum and Dogecoin)	0.29 %	4.24 %
Portfolio 4 (including ten cryptocurrencies)	-0.25 %	6.46 %

Source: compiled by the authors.

From the data in *Table 2*, it becomes clear that by including all ten cryptocurrencies studied in the portfolio of traditional assets and investing in equal parts, a higher return is not achieved, but the risk increases almost three times. Including only cryptocurrency Bitcoin in a portfolio of traditional assets achieves the highest return compared to other portfolios composed of equal parts. In this case, the inclusion of Bitcoin slightly increases the risk of the portfolio compared to an equal portfolio without cryptocurrencies.



Source: compiled by the authors.

Figure 6. Distribution of Portfolios in Equal Parts

The presented *Figure 6* clearly shows how the return and risk change with the inclusion of cryptocurrencies for all the portfolios, as well as that the inclusion of selected cryptocurrencies increases the risk of the traditional asset investment portfolio in all cases. The risk increases as more and more cryptocurrencies are added to the traditional asset investment portfolio. It is important to note that adding cryptocurrencies to a portfolio of traditional assets can worsen the return and risk of an equal portfolio.

In the research period, adding different cryptocurrencies to the portfolio of traditional assets and dividing the investment instruments in the portfolio into equal parts did not always lead to higher portfolio returns but always to increased risk. To find the most acceptable investment solution, portfolio optimisation is applied in the third stage of the research. At this stage, the most acceptable ratio of return and risk of the investment portfolio is found by applying Excel Solve and changing the parts of investment instruments in the portfolio. Additionally, portfolio optimisation is carried out based on the theories of Markowitz and

Roy. The optimisation results of the first investment portfolio consisting only of stock indices are presented in *Table 3*.

Table 3. Optimisation of the first portfolio

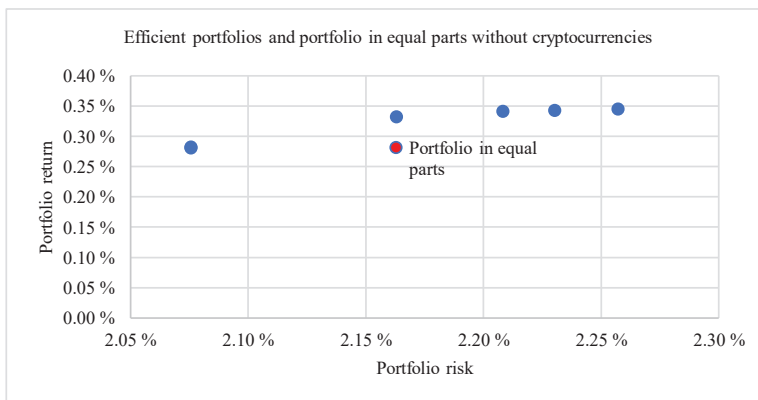
Investment instrument	Risk minimisation	Return maximisation	Maximum return	Minimum risk	$d = 0.1\%$	$d = 0.15\%$
S&P 500	35.61%	2.73%	0%	34.77%	0%	0%
Euro Stoxx 50	0%	0%	0%	0%	0%	0%
DAX	51.58%	86.70%	100%	52.48%	95.06%	97.53%
CAC 40	12.81%	10.56%	0%	12.75%	4.94%	2.47%
Indicators						
Average return	0.28%	0.33%	0.34%	0.28%	0.3407%	0.3428%
Standard Deviation/Risk	2.08%	2.16%	2.26%	2.08%	2.21%	2.23%

Source: compiled by the authors.

It can be seen from *Table 3* that by minimising risk, a lower risk of the investment portfolio was achieved with a fixed return of the equal parts portfolio. Return maximisation achieved a higher portfolio return for a given portfolio risk of equal parts. The maximum possible return and the minimum possible risk of a traditional asset portfolio were also determined. In this case, the Euro Stoxx 50 stock index was completely removed from the portfolio while searching for efficient portfolios. When determining the maximum possible return of the portfolio with unfixed risk, it is suggested to invest in only one DAX stock index.

According to Roy’s theory, the excess return and risk ratio should be maximised. The excess return is determined based on the investor’s minimum required return. It is the total expected return on investment minus the minimum return required by the investor. Each investor has a different sensitivity to risk, and therefore the required level of return d is different. The more risk-averse an investor is, the lower his required return on a risk-free investment is, that is, it approaches zero, and vice versa. *Table 3* shows that the most risk-averse investor has a possible minimum desired return of 0.1%, while a less risk-sensitive investor has a minimum desired return of 0.15%.

According to the theories of Markowitz and Roy, the distribution of efficient stock index portfolios is presented in *Figure 7*.



Source: created by the authors.

Figure 7. Distribution of Efficient Portfolios without Cryptocurrencies

Figure 7 shows that the applied portfolio optimisation helped identify efficient stock index portfolios and find the optimal combinations of return and risk for the investment portfolio.

The optimisation results obtained and the analysis of the remaining three investment portfolios with different cryptocurrencies are presented below. The optimisation results of the second investment portfolio, including the cryptocurrency Bitcoin, are presented in Table 4.

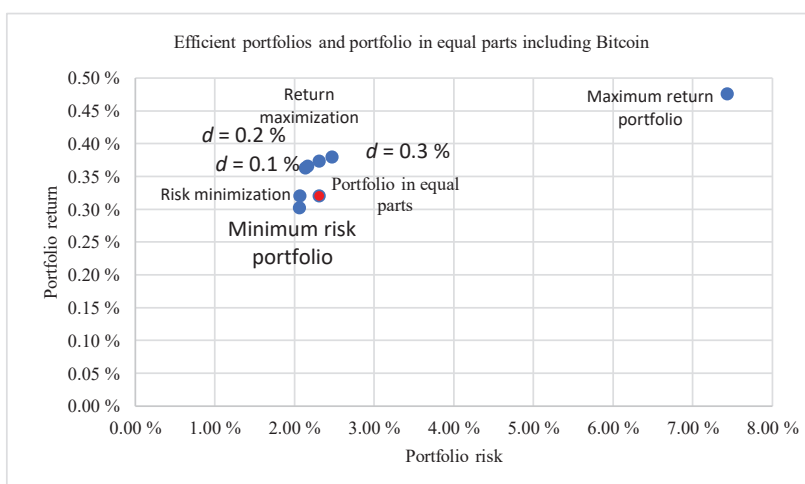
Table 4. Optimisation of the second portfolio

Investment instrument	Risk minimisation	Return maximisation	Maximum return	Minimum risk	$d = 0.1\%$	$d = 0.2\%$	$d = 0.3\%$
S&P 500	21.35%	0%	0%	25.99%	0%	0%	0%
Euro Stoxx 50	0%	0%	0%	0%	0%	0%	0%
DAX	65.86%	78.32%	0%	50.86%	86.50%	84.20%	73.95%
CAC 40	4.64%	0%	0%	15.89%	0%	0%	0%
Bitcoin	8.15%	21.68%	100%	7.26%	13.50%	15.80%	26.05%
Indicators							
Average return	0.32%	0.37%	0.48%	0.30%	0.36%	0.37%	0.38%
Standard Deviation / Risk	2.07%	2.31%	7.44%	2.07%	2.14%	2.17%	2.47%

Source: compiled by the authors.

From the presented Table 4, it can be seen that applying portfolio optimisation to the portfolio of traditional assets, including Bitcoin cryptocurrency, reduced the risk with fixed return of the second equal parts portfolio and increased the return with fixed risk of the second equal parts portfolio. After the second portfolio optimisation, the Euro Stoxx 50 was removed from the investment portfolio in all cases. The S&P 500, CAC 40 stock indices were mostly removed from the investment portfolio. In the case of maximum returns, there is no more diversification of the portfolio and it is suggested to invest only in the cryptocurrency Bitcoin.

As mentioned earlier, every investor has a different sensitivity to risk. From the data in Table 4, it is clear that the less risk sensitive an investor is, the higher the return and risk of his investment portfolio with Bitcoin. It is also observed that the part of included cryptocurrency in the portfolio increases as the investor's sensitivity to risk decreases.



Source: created by the authors.

Figure 8. Distribution of Efficient Portfolios, Including Bitcoin

Figure 8 shows in detail the distribution of the efficient stock index portfolios with the inclusion of the Bitcoin cryptocurrency.

Figure 8 shows that the optimisation improved the return-risk combination compared to the equal parts portfolio. Minimising risk at a given equal parts second portfolio return shifted the investment portfolio to the left while maximising returns at a given equal parts second portfolio risk shifted the investment portfolio up. Additionally, *Figure 8* clearly shows the efficient portfolios identified with the required risk-free return specified by the investor.

The results of the return and risk optimisation of the third investment portfolio consisting of stock indices, including the cryptocurrencies Bitcoin, Ethereum and Dogecoin are presented in *Table 5*.

Table 5. Optimisation of the third portfolio

Investment instrument	Risk minimisation	Return maximisation	Maximum return	Minimum risk	$d = 0.1\%$	$d = 0.2\%$	$d = 0.3\%$
S&P 500	29.38%	0%	0%	25.99%	0%	0%	0%
Euro Stoxx 50	0%	0%	0%	0%	0%	0%	0%
DAX	39.87%	43.68%	0%	50.86%	86.50%	84.20%	73.95%
CAC 40	24.14%	0%	0%	15.89%	0%	0%	0%
Bitcoin	6.61%	56.32%	100%	7.26%	13.50%	15.80%	26.05%
Etherium	0%	0%	0%	0%	0%	0%	0%
Dogecoin	0%	0%	0%	0%	0%	0%	0%
Indicators							
Average return	0.29%	0.42%	0.48%	0.30%	0.36%	0.37%	0.38%
Standard Deviation / Risk	2.07%	4.24%	7.44%	2.07%	2.14%	2.17%	2.47%

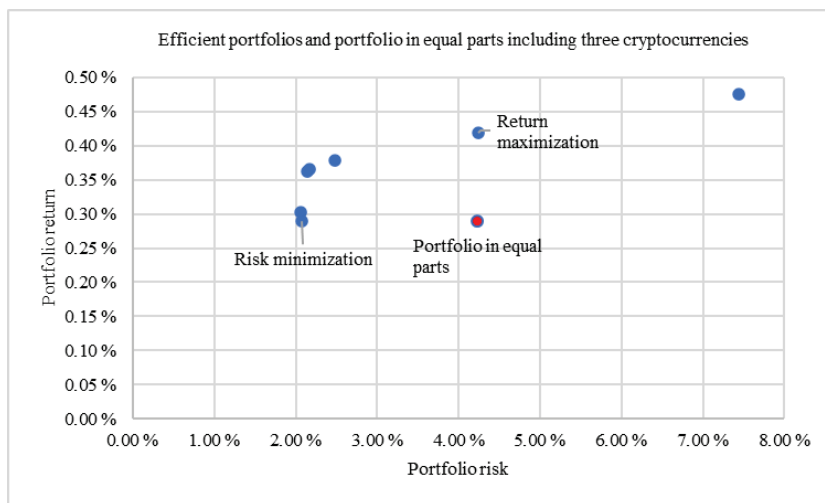
Source: compiled by the authors.

From the results of *Table 5*, it can be seen that in all the cases during optimisation, Bitcoin was left in the investment portfolio, while Ethereum and Dogecoin were removed from the investment portfolio in all the cases. In search of the maximum return of the third investment portfolio with an unfixed risk, there is no more portfolio diversification and it is suggested to invest only in the cryptocurrency Bitcoin.

As seen from *Table 5*, as the investor's sensitivity to risk decreases, the part of Bitcoin in the portfolio increases. In most cases, efficient investment portfolios include the DAX stock index and the Bitcoin cryptocurrency. As in the case of the second portfolio, the Euro Stoxx 50 was excluded in all cases and the S&P 500, CAC 40 stock indices were excluded in most cases when identifying the effective investment portfolios of the third portfolio.

The distribution of the efficient portfolios identified in the third investment portfolio, including three cryptocurrencies, is presented in *Figure 9*.

As already mentioned, the cryptocurrencies Ethereum and Dogecoin were removed from the efficient portfolios in all cases. *Figure 9* clearly shows that as the expected return of the investment portfolio increases, so does the risk. When comparing the identified efficient portfolios with the portfolio made up of equal parts, it can be seen that efficient portfolio return and risk combinations were found during optimisation.



Source: created by the authors.

Figure 9. Distribution of Efficient Portfolios, Including Three Cryptocurrencies

The results of the return and risk optimisation of the fourth investment portfolio consisting of stock indices, including ten cryptocurrencies, are shown in Table 6.

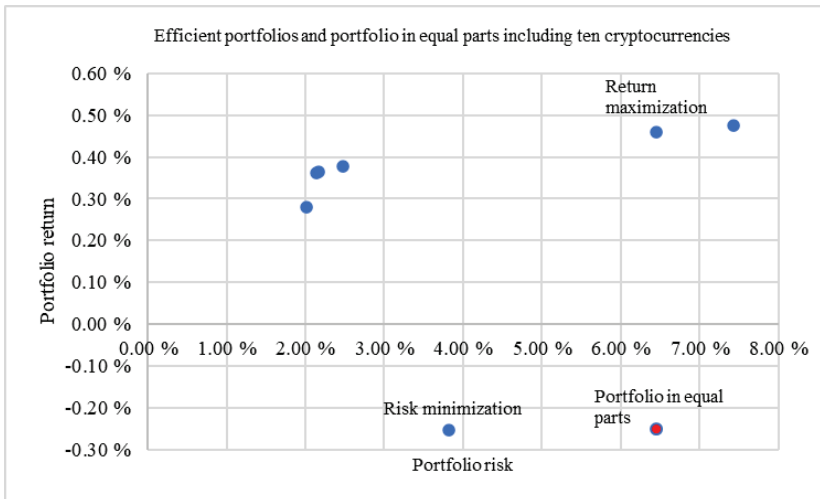
Table 6. Optimisation of the fourth portfolio

Investment instrument	Risk minimisation	Return maximisation	Maximum return	Minimum risk	$d = 0.1\%$	$d = 0.2\%$	$d = 0.3\%$
S&P 500	58.72%	0%	0%	29.09%	0%	0%	0%
Euro Stoxx 50	0%	0%	0%	0%	0%	0%	0%
DAX	0%	13.05%	0%	46.34%	86.50%	84.20%	73.95%
CAC 40	9.64%	0%	0%	11.71%	0%	0%	0%
Bitcoin	0%	86.95%	100%	0.88%	13.50%	15.80%	26.05%
Ethereum	0%	0%	0%	0%	0%	0%	0%
Binance Coin	2.95%	0%	0%	0%	0%	0%	0%
Dogecoin	0%	0%	0%	0%	0%	0%	0%
Cardano	0.01%	0%	0%	0%	0%	0%	0%
Solana	0%	0%	0%	0%	0%	0%	0%
TRON	0%	0%	0%	11.96%	0%	0%	0%
Polkadot	9.70%	0%	0%	0%	0%	0%	0%
Shiba Inu	0%	0%	0%	0%	0%	0%	0%
Avalanche	18.99%	0%	0%	0%	0%	0%	0%
Indicators							
Average return	-0.25%	0.46%	0.48%	0.28%	0.36%	0.37%	0.38%
Standard Deviation / Risk	3.83%	6.46%	7.44%	2.01%	2.14%	2.17%	2.47%

Source: compiled by the authors.

The data in Table 6 shows that, in most cases, only Bitcoin remains in the efficient portfolios of the included cryptocurrencies. Cryptocurrency Bitcoin was removed only in risk minimisation at the fixed return of equal parts of the fourth portfolio. However, it is important to note that the return is negative in this case. It can also be seen that the smallest part is allocated to the cryptocurrency Bitcoin to ensure minimum risk in the investment portfolio. In this case, a larger part of the investment portfolio is allocated to the cryptocurrency TRON.

As in the case of the second and third portfolios, the Euro Stoxx 50 and, in most cases, the S&P 500, CAC 40 stock indices were excluded from the identification of the efficient investment portfolios of the fourth portfolio. For an investor who is less sensitive to risk, efficient investment portfolios consist only of the cryptocurrency Bitcoin and the DAX stock index.



Source: created by the authors.

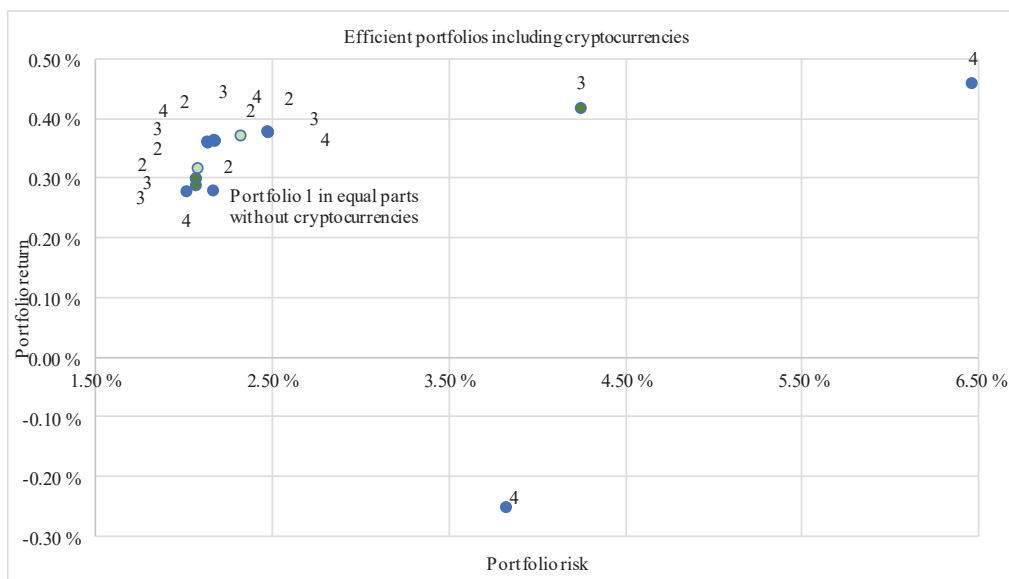
Figure 10. Distribution of Efficient Portfolios, including Ten Cryptocurrencies

The distribution of the identified efficient portfolios of the fourth investment portfolio, including ten cryptocurrencies, is presented in Figure 10.

When analysing the data presented in Figure 10, it can be seen that optimisation improved the portfolio return at the fixed risk of the equal parts portfolio, as well as reduced the risk at the fixed return of the equal parts portfolio. In this case, it can be said that to create an efficient investment portfolio, it is appropriate to include the cryptocurrency Bitcoin.

In order to generalise the analysis of efficient portfolios that included different cryptocurrencies, efficient portfolios that did not exclude all cryptocurrencies or all stock indices during the optimisation were looked at in more detail. The distribution of all these portfolios is presented in Figure 11.

When summarising the obtained data, it can be stated that in order to achieve the maximum investment return without fixing portfolio risk, portfolio diversification is no longer possible in all cases and it is suggested to invest in only one investment instrument. In this case, when optimising the second, third and fourth portfolios, it is suggested to invest only in the cryptocurrency Bitcoin. The fourth portfolio, which included all ten studied cryptocurrencies, stood out the most among all analysed portfolios. Including all ten cryptocurrencies in a traditional asset investment portfolio and allocating equal parts to all investment instruments resulted in negative returns and increased portfolio risk compared to other portfolios with equal allocations. Figure 11 shows that applying portfolio optimisation to reduce the risk of the fourth portfolio at a fixed equal portfolio return resulted in a lower risk for this portfolio.



Source: created by the authors.

Figure 11. Distribution of Efficient Portfolios, including Cryptocurrencies

The presented *Figure 11* shows that in most cases, the effective investment portfolios of the second, third, and fourth portfolios are distributed similarly. In most cases, the cryptocurrency Bitcoin remained in the efficient portfolios. Cryptocurrencies Ethereum, Dogecoin, Solana and Shiba Inu were removed entirely from efficient portfolios during optimisation. When summarising the data, it can often be observed that the lower the risk in the portfolio, the smaller the part of cryptocurrency in the portfolio. As the target return increases or the investor becomes less sensitive to risk, in most cases, the part of included stock indices decreases and the part of cryptocurrencies in the portfolio increases. During optimisation, the EuroStoxx 50 stock index was excluded from the investment portfolio in all cases. Most often, the DAX stock index remained in effective investment portfolios along with included cryptocurrencies. Less often – the S&P500 and CAC40 stock indices.

Conclusions

After conducting an analysis of the scientific literature and summarising the portfolio theory analysed in the literature, it can be concluded that the benefits provided by the Markowitz model are apparent, and the most important advantages of the theory are the distinguished benefits of diversification, the management of the overall risk of the portfolio and the possibility of achieving the highest expected return with the lowest possible risk. For this reason, portfolio theory could be useful in researching the inclusion of cryptocurrencies in an investment portfolio, as it provides the opportunity to build an efficient portfolio of risky assets with the lowest possible risk. The main benefit of cryptocurrencies when adding them to a portfolio is diversification, but it is a difficult task for investors to invest with expected returns due to the high volatility of cryptocurrencies. Including cryptocurrencies in an investment portfolio can have a positive effect on its efficiency, and the risks associated with

cryptocurrencies can be compensated by the corresponding returns. However, it is important to note that real investment decisions should be based on market scenarios and verified.

After establishing the research methodology, it can be stated that the applied portfolio theories of Markowitz and Roy allow for determining the allocation of resources between investment instruments and finding efficient portfolios, i.e. the most appropriate or acceptable combinations of expected return and risk for the investor and to present conclusions on the effectiveness of including cryptocurrencies in the investment portfolio.

The research data obtained allow us to conclude that in order to achieve efficiency in the investment portfolio, the inclusion of not all ten cryptocurrencies in the investment portfolio may be beneficial. To achieve the maximum return on the investment portfolio, there is no more diversification in the portfolio in the presence of unfixed risk. As the desired return increases or the investor is less sensitive to risk, a decrease is observed in the part of included stock indices and an increase in the part of cryptocurrencies in the portfolio.

After identifying efficient investment portfolios in the research, it can be concluded that the inclusion of different cryptocurrencies in an investment portfolio consisting of stock indices does not always allow to achieve efficient portfolio results. According to the research results, the inclusion of Bitcoin in the portfolio of traditional assets affects its efficiency; because of the ten cryptocurrencies studied, the inclusion of Bitcoin in the investment portfolio in most cases led to efficient portfolio results. Here, it is necessary to note that for the sake of efficiency, the portfolio of stock indices in which cryptocurrencies are included should be composed of the S&P500, CAC40 and DAX stock indices.

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INVESTICIJŲ PORTFELIO EFEKTYVUMO TYRIMAS ĮTRAUKIANT SKIRTINGAS KRIPTOVALIUTAS

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SANTRAUKA

Technologijų plėtra ir globalizacija atvėrė plačias galimybes net ir atitinkamų žinių ar patirties neturintiems investuotojams naudotis investicinėmis priemonėmis. Nors kriptovaliutas turi unikalių savybių, kurios traukia investuotojus, tačiau rinkai būdinga ir rizika: kainų svyravimai, operacinė rizika ir pasitikėjimo problema. Šiame kontekste ieškoti efektyvių investicijų portfelių sprendimų ir gebėti geriau suprasti bei vertinti kriptovaliutų rinką tampa ypač svarbu. Ankstesniuose tyrimuose pabrėžiama kriptovaliutų teikiama nauda yra diversifikavimas, apsidraudimas ir saugus prieglobstis, tačiau mažai yra tyrimų, atskleidžiančių kriptovaliutų portfelio rezultatus. Investuotojai siekia kuo didesnės grąžos, bet paprastai vengia ar neįvertina rizikos. Tyrime pritaikius portfelio optimizavimą siekiama identifikuoti kriptovaliutas, kurių įtraukimas į tradicinio turto investicijų portfelį padėtų pasiekti efektyviausią siekiamos grąžos ir rizikos derinį. Pagrindinis šio tyrimo tikslas yra įvertinti skirtingų kriptovaliutų įtaką investicijų portfelio efektyvumui. Tiriamos dešimt kriptovaliutų, įeinančių į B10 BITA CRYPTO 10 INDEX sudėtį. Dėl kriptovaliutų nepastovumo šio indekso sudėtis nuolat keičiasi. Tyrime analizuojamos kriptovaliutos, įeinančios į šio indekso sudėtį, buvo fiksuotos 2023 m. rugpjūčio mėn. Tradicinį turimą tyrime atspindi akcijų indeksai: S&P 500, Euro Stoxx 50, DAX ir CAC 40. Tyrime identifikavus efektyvius investicijų portfelius galima daryti išvadą, kad skirtingų kriptovaliutų įtraukimas į investicijų portfelį, sudarytą iš akcijų indeksų, ne visada leidžia pasiekti efektyvius portfelio rezultatus.

REIKŠMINIAI ŽODŽIAI: akcijų indeksai; diversifikacija; efektyvus portfelis; kriptovaliutos; portfelio optimizavimas; tradicinis turtas.