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The passivity of adaptive output regulation of nonlinear exosystem with application of aircraft motions

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Abstract. This paper deals with passivity of adaptive output regulation of nonlinear exosystem. It is shown that factorisable low-high frequency gains and harmonic uncertainties are estimated to the exogenous signals with adaptive nonlinear system. The design methodology guarantees asymptotic regulation in the case where the dimension of the regulator is sufficiently large in relation, which affects the number of harmonics acting on the system. On the other hand, harmonics of uncertain amplitude, phase, and frequency are the major sources, and the bounded steady-state regulation error ensures that adaptive nonlinear system is globally asymptotically stable via passivity theory. Kalman–Yacubovitch–Popov property provides that the uncertain adaptive nonlinear system is passive. Finally, specific examples are shown in order to demonstrate the applicability of the result.

Keywords: Lyapunov function, output regulation, passivity.

1 Introduction

It is well known that the passivity theory of deterministic nonlinear systems was first founded by [21], which is a powerful technique in handling stability issue and has important application in many engineering problems. The nonlinear dissipative control and

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passivity have showed that some inequality involves supply rates and storage functions [4, 6, 17]. Especially, Hill and Moylan [6] proposed a nonlinear version of the Kalman–Yacubovitch–Popov (KYP) property and a sufficient and necessary condition for an affine nonlinear system to be passive. The principal imposed that the state equations should involve the control vector, which is only linear. In [16], multivalued controls are derived from a special maximally monotone operator, and they suggested that strongly passive linear system (with possible parametric uncertainty and external disturbances) with multivalued control laws are ensuring regulation of the output to be desired value. It can be shown that, for mechanical systems, the energy balancing approach and energy shaping techniques are used port-Hamiltonian passivity with output regulation system. Our methodology is different from port-Hamiltonian passivity with output regulation setting. If an internal model with transfer function is in the feedback loop and the closed-loop system is stable, then we obtain tracking and/or disturbance rejection for sinusoidal reference, and disturbance signals of frequency gain exists in exosystem. If the reference and disturbance signals are periodic, then the internal model principle leads to repetitive control. The objective of our study is exosystem with internal model minimum phase based on n regulated outputs that are converted as single input nonlinear adaptive formation, with support of KYP property, this exosystem is derived for passivity.

In [2], passivity combined with geometric nonlinear control theory has been proved. The results developed in [2], the global stabilization of nonlinear systems, robust and adaptive control of minimum phase nonlinear systems, found out parametric uncertainty. The aim of passivity-based synthesis approach controller is to render a nonlinear system to be passive. But in this, any information for nonlinear systems with structural uncertainties or uncertain perturbations were not given. One of the main motivations for studying passivity in control theory context is to prove the stability and structural uncertainties. An important result in this area is KYP lemma, which is used in solving the well-known stability problems. However, these known results are related only to the case of state feedback passivity. In [4, 10, 17, 18], the view of input-output nature of the passivity concept is described, and it seems useful to establish relations between output feedback counterparts of stability and passivity. All the above literature are studied continuous time control schemes with properties of feedback equivalent to passive, it is particular class of interconnected internal model with zero-dynamics, i.e., weak minimum phase condition, but not sufficient for feedback equivalence to nonlinear passive systems. However, as far as we know, a general nonlinear system with output regulation to be passive is still remains open and challenging. The nonlinear system with output regulation of passivity is useful in aircraft handling qualities.

The aircraft motion is regulated by line-of-sight (LOS) angle rates, which will be regulated in some range, and those rates are commanded in aircraft maneuvering [9]. The LOS angles are related to output regulation of nonlinear systems [9, 11]. It is showed; aircraft with follower aircraft communication error signals are interrelated to output regulation minimum phase nonlinear system. These concepts are more useful to design the autopilot, state feedback controller based eigen structure assignment methods, and those are commonly used to aircraft handling qualities etc. The problem of nonlinear systems with output regulation is solved by variable gain feedback law that is more similar to adaptive learning regulation of uncertain minimum phase systems [1,3,5,7,12–14,19]. In this paper, the variable $\mathbf{w}$ is an exogenous variable, that is modeling reference has to be tracked and or disturbance has to be rejected. The exogenous $\mathbf{w} = \mathrm{col}(\omega, w)$ will be taken as n harmonic oscillators with low-high frequency gain, where $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ are output feedback error disturbances and $w = (w_1, w_2, \dots, w_m)$ are taken as estimated disturbances. The variable $\dot{w} = s(w) = \left(\begin{smallmatrix} w & 0 \\ 0 & -w \end{smallmatrix}\right)$, where w are over-estimated disturbances, $-w$ are under-estimated disturbances. And in this case, under-estimated disturbances will be dominated. The use of conditional servo compensator enables to achieve zero steady-state tracking error without degrading the transient response of the system; it is given in [15]. In [20], the problem of robust adaptive output regulation of hybrid adaptive external systems is described by nonlinear differential equations. In those article, information for under-dimensional internal model is not given. It was shown that set of all feed-forward inputs are essential to keep the tracking error identically at zero, which is a subset of solutions of a linear differential equation.

In recent work [8], studied Rayleish oscillator as a back born of aircraft maneuverability. The exogenous signals are transformed to interconnected subsystem with appropriate structures, i.e., one subsystem contains no control law, and whereas the other one is regulated error output is a control input, which satisfies a Lyapunov inequality on behalf of weak minimum phase condition. In particular, in [8], imposed adaptive output regulation for nonlinear systems in the case of exosystem signals with harmonic oscillators was emphasized, it does not give any information about control output equivalence to feedback control. Compared with deterministic case, up to date, there still requires much work of investigating the nonlinear system with output regulation of minimum phase exosystem is global asymptotic stabilization. Based on the above, we investigate global asymptotic stabilization the relationship between output regulation of passive system and corresponding minimum phase of adaptive nonlinear systems. Especially, different from the deterministic exosystem, our results first show that the local asymptotic stability of non zero-dynamics systems under the low-gain frequency gain. Then, sufficient conditions derived for exosystem with n regulated outputs are converted as single input adaptive nonlinear form, and global asymptotic stability is provided via passivity theory. Finally, the physical example aircraft motion shows the applicability of the obtained results.

The paper is organized as follows. Section 2 contains some basic assumptions and problem formulations. The main results and discussion are given in Section 3. In Section 4, the examples are considered, and in Section 5 some conclusions are drawn.

2 Problem formulation and preliminaries

Consider the nonlinear system

$$\dot{z} = f(\mathbf{w}, z, e), \tag{1}$$

$$\dot{e}_i = e_{i+1}, \quad i = 1, \dots, r-1, \tag{2}$$

$$\dot{e}_r = q(\mathbf{w}, z, e) + u, \tag{3}$$

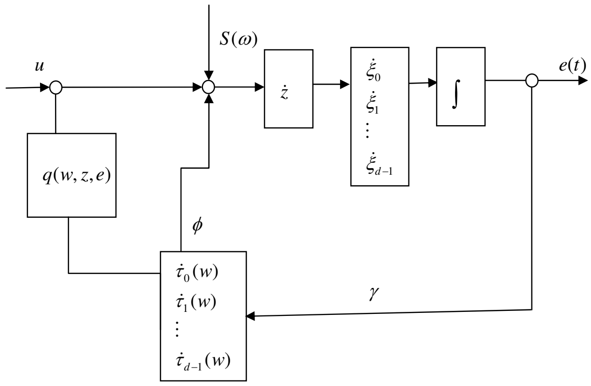


Figure 1. The control scheme of formulated system (1)–(3).

where $z \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the control input, the regulated output $e = (e_1, \dots, e_r) \in \mathbb{R}^r$, and the unmeasured input $\mathbf{w} \in \mathbf{W}$ is an exogenous signal with r harmonic oscillators, that is supposed to be generated by the smooth exosystem

$$\dot{\mathbf{w}} = s(\mathbf{w}), \tag{4}$$

which is invariant at $\mathbf{W} \subset \mathbb{R}^{d\times d}$, where $s(\mathbf{w}) = \left(\begin{smallmatrix} \omega & 0 \\ 0 & \omega \end{smallmatrix}\right)\left(\begin{smallmatrix} w & 0 \\ 0 & -w \end{smallmatrix}\right)$ and $\mathbf{w} = \mathrm{col}(\omega, w) \in \mathbf{W}$. Assume that the set $\mathbf{W} \subset \mathbb{R}^{d\times d}$ is admissible initial conditions for exosystem (4), which is a compact set, where $\mathbf{W} = \bigcup_{\omega, w \in \mathbb{R}^{d\times d}} \left(\begin{smallmatrix} \omega & \omega \\ \omega & \omega \end{smallmatrix}\right) \times \left(\begin{smallmatrix} w & w \\ -w & -w \end{smallmatrix}\right)$ and $\omega = (\omega_1, \omega_2, \dots, \omega_n)$, $w = (w_1, w_2, \dots, w_m)$. In this case, $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ are n harmonic components of unknown frequency dependent on unknown amplitudes and phases dependent on the initial condition of w. Let $\mathbf{W} \times Z \times E \subset \mathbb{R}^{d\times d} \times \mathbb{R}^n \times \mathbb{R}^r$ be a compact of initial state of (1)–(3) for which problem (1)–(3) is solvable when assuming that trajectories of the zero dynamics of (1)–(3) augmented with (4) are bounded.

Notation. In this paper, $\mathbb{R}$ denotes the real numbers, C^1 is continuous differentiable on some closed time interval. For any $x \in \mathbb{R}^n$, $|x|$ denotes the Euclidean norm of $\mathbb{R}^n$. G is the vehicle's center of mass, m is its mass, J denotes inertia matrix, and $\mathcal{I}$ denotes inertial frame with respect to vehicles absolute position is measured.

It follows from the above assumption $s(\mathbf{w}) = \left(\begin{smallmatrix} \omega & 0 \\ 0 & \omega \end{smallmatrix}\right)\left(\begin{smallmatrix} w & 0 \\ 0 & -w \end{smallmatrix}\right)$ is a nonempty, compact invariant set, which is stable in the sense of Lyapunov and uniformly attracts $\mathbf{W} \times Z \times E$. If there exist an integer p and a locally Lipschitz function $f : \mathbb{R}^p \to \mathbb{R}$ such that, for any $(\mathbf{w}, z_0, \varphi) \in \mathbf{W} \times Z \times E$, the solution $(\mathbf{w}, z, \varphi)$ of (1)–(3) passing through $(\mathbf{w}, z_0, \varphi)$ at $t = 0$, then such the function $\varphi = -q(\mathbf{w}, z, 0)$ satisfies $\varphi^p + f(\varphi, \varphi^{(1)}, \varphi^{(2)}, \dots, \varphi^{(p-1)}) = 0$.

Furthermore, system (1)–(3) exists in pair sets $\mathbf{W} \times Z \times E \subset \mathbb{R}^{d\times d} \times \mathbb{R}^n \times \mathbb{R}^r$ and $\Theta \subset \mathbb{R}^p$ (the latter is a compact set for any integer p), then the error feedback controllers

$$\dot{\xi} = \alpha(\xi, e), \qquad u = \beta(\xi, e) \tag{5}$$

and the initial conditions are in a compact set $\mathbf{W}\times Z\times\Theta\times E\subset\mathbb{R}^{d\times d}\times\mathbb{R}^n\times\mathbb{R}^p\times\mathbb{R}^r$ such that trajectory of resulting closed-loop system (1)–(4) originating from $\mathbf{W}\times Z\times\Theta\times E$ are bounded and $\lim_{t\to\infty}e(t)=0$.

The problem of semi global output regulation in which $r=1$ and assume the change of variable $e_i\to\varphi=k^{-(i-1)}e_i$, $i=1,\dots,r-1$, such that system (1)–(3) can be written as

$$e_i\to\theta=e_r+k^{(r-1)}a_0e_1+k^{(r-2)}a_1e_2+\dots+ka_{(r-2)}e_{r-1}$$

in which $k>1$ is a design parameter and the a_i, $i=0,\dots,r-2$, are roots of the polynomial $\lambda^{r-1}+a_{r-2}\lambda^{r-1}+\dots+a_1\lambda+a_0=0$ have a negative real parts. The change of variables can be applied for (1)–(3), we have

$$\begin{aligned}&\dot{\mathbf{w}}=s(\mathbf{w}),\qquad \dot{z}=f(\mathbf{w},z,\varphi)\\&\dot{\varphi}=kA_H\varphi+B\theta,\qquad \dot{\theta}=\overline{q}(\mathbf{w},z,\varphi,\theta,k)+\overline{b}(\mathbf{w},z,\varphi,\theta,k)u,\end{aligned}\tag{6}$$

where $\varphi=(\varphi_1,\varphi_2,\dots,\varphi_{r-1})$, A_H is Hurwitz matrix, and $\overline{q},\overline{b}$ are smooth functions, $\overline{b}(\mathbf{w},z,\varphi,\theta,k)\geqslant\overline{b}$, $(\mathbf{w},z,\varphi,\theta)\in\mathbf{W}\times\mathbb{R}^n\times\mathbb{R}^{r-1}\times\mathbb{R}$ and for all $k>0$.

If $\varphi_1=e_1$, which is exists in a compact set E, system (6) regarded as a system with input u and output θ have relative degree one and zero dynamics, then

$$\dot{\mathbf{w}}=s(\mathbf{w}),\qquad \dot{z}=f(\mathbf{w},z,\varphi_1),\qquad \dot{\varphi}=A_H\varphi.\tag{7}$$

Under the condition $\mathrm{grap}(\pi)=\{(\mathbf{w},z)\in\mathbf{W}\times\mathbb{R}^n\colon z=\pi(\mathbf{w})\}\times\{0\}$, the classical result [2] showed that system (7) is globally asymptotically stable.

In this setting, we assume that the controller

$$\dot{\xi}=\alpha(\xi,\theta),\qquad u=\beta(\xi,\theta)\tag{8}$$

solves the problem of output regulation (7). This controllers are driven by the "dummy" regulated output θ, and it is not actual regulated output φ_1. However, in this case, θ can be constructed as a linear components of $\varphi_1,\varphi_2,\dots,\varphi_r$, which is related to the partial state $e=(e_1,e_2,\dots,e_r)$. If φ coincides with $(r-1)$th derivative with respect to time, there is an actual regulated output e.

As it is well known, $e_1,e_2,\dots,e_r$ purpose is to convergence to desired set, it can be replaced by appropriate estimates γ. Using these estimates to replace the expression of θ in (8) yields a controller able to solve the problem for the original plant (1)–(3). On the basis of these arguments, the design of controller (8) with system (1)–(4) can be solved, and the solutions of the output regulation are obtained by $\pi:\mathbb{R}^{d\times d}\to\mathbb{R}^{n\times 1}$. If there exists a continuous differential function $\pi:\mathbf{W}\to\mathbb{R}^{n\times 1}$, then $\pi(\mathbf{w})=-q(\mathbf{w},z,0)$ with $\pi(0)=0$ be a solution of the regulation equation (1)–(3) such that

$$\frac{\partial\pi(\mathbf{w})}{\partial\mathbf{w}}s(\mathbf{w})=f\bigl(\mathbf{w},z(\mathbf{w}),0\bigr)\quad\forall\mathbf{w}\in\mathbf{W}.\tag{9}$$

If we assume that $F:\mathbb{R}^{p\times 1}\to\mathbb{R}^{p\times 1}$ be a locally Lipschitz function, $\gamma:\mathbb{R}^{p\times 1}\to\mathbb{R}$ be a continuous function, the column vector $G\in\mathbb{R}^{p\times 1}$, the regulator $u^*=-q(\mathbf{w},\pi(\mathbf{w}),0)$, then the continuous differential function τ is mapping from $\tau : \mathbf{W} \to \mathbb{R}^{p\times 1}$ such that

$$\frac{\partial \tau}{\partial \mathbf{w}} s(\mathbf{w}) = F\big(\tau(\mathbf{w})\big) + G\big(\gamma\big(\tau(\mathbf{w})\big)\big), \qquad u^* = \gamma\big(\tau(\mathbf{w})\big), \tag{10}$$

$$\dot{\mathbf{w}} = s(\mathbf{w}), \qquad \dot{\xi} = F(\xi) + Gu^*(\mathbf{w}) \tag{11}$$

is locally asymptotically stabilizable in $\{(\mathbf{w}, \xi) \in \mathbf{W} \times \mathbb{R}^{p\times 1}\colon \xi = \tau(\mathbf{w})\} = \Omega$ with domain of attraction $\mathbf{W} \times \mathcal{A}$, where $\mathcal{A}$ is an open set of $Z \times \Theta \times E$ ($\mathcal{A} \supset Z \times \Theta \times E$).

Let $\tau(\mathbf{w})$ be a solution of nonlinear differential equation (1)–(3), which exists in exosystem such that $\tau_0(\mathbf{w}) = u^*(\mathbf{w})$, $\tau_{i+1}(\mathbf{w}) = \partial L_p^i u^*(\mathbf{w})$ for $i = 0, 1, \dots, p-1$, and $\tau(\mathbf{w}) = \mathrm{col}(\tau_0(\mathbf{w}), \dots, \tau_{p-1}(\mathbf{w}))$. If $\tau(\mathbf{w})$ is large enough and satisfies the above conditions, then $u^*(\mathbf{w})$ is well-defined in C^1. If feed-forward $\phi = u^*(\mathbf{w})$ such that ϕ will be a solution of nonlinear system (1)–(3), which is exists in $\mathcal{A}$ and $\phi^p + g(\phi^{(p-1)}, \dots, \phi^1, \phi) = 0$. This details can be found in the literature [7–9].

The following theorem shows that how to solve the problem of output regulation (1)–(4) in an appropriate domain.

Theorem 1. *Suppose that $F(\cdot)$, $G(\cdot)$, $\gamma(\cdot)$) are fulfilling* (10) *and τ and ξ are locally asymptotically stablilizable in Ω, then there exists a continuous function $k : \mathbb{R} \to \mathbb{R}$ such that the controller*

$$\dot{\xi} = F(\xi) + G\big(v + \gamma(\xi)\big), \qquad u = \gamma(\xi) + v, \qquad v = -k(e) \tag{12}$$

solves the problem of output regulation (1)–(4) *in $\mathbf{W} \times \mathcal{A} \subset \mathbf{W} \times Z \times \Theta \times E$.*

More conventital design and methodologies has been obtained from Marconi et al. [8, 9]. In that design, the triplet $(F(\cdot), G(\cdot), \gamma(\cdot))$ have been fulfilling internal model property, and it can be effectively carried out in the case of $\phi : \mathbb{R}^{p\times 1} \to \mathbb{R}$ such that

$$L_p^p u^*(\mathbf{w}) = \phi\big(u^*(\mathbf{w}), L_p u^*(\mathbf{w}), \dots, L_p^{p-1} u^*(\mathbf{w})\big)$$

for all $\mathbf{w} \in \mathbf{W}$. In this setting,

$$\tau(\mathbf{w}) = \big(\tau_0(\mathbf{w}),\, \dots,\, \tau_{p-1}(\mathbf{w})\big)^{\mathrm{T}} = \big(u^*(\mathbf{w}),\, \dots,\, L_p^{p-1} u^*(\mathbf{w})^{\mathrm{T}}\big), \tag{13}$$

and let ϕ_c be any locally Lipschitz and bounded function such that ϕ_c in $\tau(\mathbf{w})$. It carry out that

$$F(\xi) = \big(\xi_1,\, \dots,\, \xi^{p-1},\, -\phi_c(\xi_1, \dots, \xi_{p-1})\big)^{\mathrm{T}} - G\xi_0, \tag{14}$$

where G be any vector, $\xi = (\xi_0, \dots, \xi_{p-1})$ and makes (12) fulfilled with $\gamma(\xi) = \xi_0$. If $G = (g\lambda_0, g^2\lambda_1, \dots, g^p\lambda_{p-1})$, where $(\lambda_0, \lambda_1, \dots, \lambda_{p-1})$ are coefficients of a Hurwitz polynomial, then $g > 0$ be a high-gain observer such that (12) is locally exponentially stabilizable in Ω with domain of attraction $\mathbf{W} \times \mathcal{A}$.

We need the following assumptions to prove that system (1)–(3) is passive.

Assumption 1. For any compact subset $\Omega \subset \mathbb{R}^{n\times 1} \times \mathbf{W}$, there exists a C^1 function V_z satisfying $\rho_1(\|z\|) \leqslant V_z \leqslant \rho_2(\|z\|)$ such that, for any $\mathbf{w} \in \mathbf{W}$, along with trajectory of system (1)–(3) has $\dot{V}_z \leqslant -\rho(\|z\|) + \delta\iota(e)$, where ρ, $\rho_1(\cdot)$ and $\rho_2(\cdot)$ are some known class κ functions and satisfies $\lim_{t\to\infty} \sup \rho^{-1} t < \infty$. δ is some unknown positive constant, and $\iota(e)$ is a known smooth positive definite function.

Assumption 2. There exist the smooth functions q, q_{11}, q_{12}, $q(w,0,0) = 0$ that are locally Lipschitz and bounded for all $M_0, M_1, M_2 > 0$ such that

$$\begin{gathered} \left|q_1(\mathbf{w},z,e) - q_1(\mathbf{w},z,0)\right| \leqslant M_0|e|, \\ \left|q_{11}(\mathbf{w},z,e)\right| \leqslant M_1, \qquad \left|q_{12}(\mathbf{w},z,e)\right| \leqslant M_2. \end{gathered}$$

Assumption 3. There exist the smooth functions $\overline{f}_{11}$, $\overline{f}_{12}$ that are locally Lipschitz and bounded for all $N_1, N_2 > 0$ such that

$$\left|\overline{f_{11}}(\mathbf{w},z,e)\right| \leqslant N_1 \quad \text{and} \quad \left|\overline{f}_{12}(\mathbf{w},z,0)\right| \leqslant N_2.$$

Assumption 4. There exist a smooth function $\overline{k_1}$, $\overline{k}_1(0) = 0$, that are locally Lipschitz and bounded for all $K_0 > 0$ such that $|\overline{k}_1(e)| \leqslant K_0$.

Remark 1. Suppose that Assumption 1 is fulfilled, and system (1)–(3) is input-to-state stable with respect to state z, the input u and equilibrium at $z = 0$ of the system $\dot{\mathbf{w}} = s(\mathbf{w})$, $\dot{z} = f(\mathbf{w},z,0)$ are locally exponentially stable for any $\mathbf{w} \in \mathbf{W}$ if ρ_1 and ρ_2 are locally quadratic.

Theorem 2. *Consider a system described by equations of the form*

$$\dot{\mathbf{w}} = s(\mathbf{w}), \qquad \dot{\xi} = F(\xi) + G\big(v + \gamma(\xi)\big), \qquad u = \gamma(\xi) + v, \qquad v = -k(e)$$

in which $F(\xi)$ and $G(u)$ are smooth vector fields and $F(0) = 0$. If there exist the smooth feedback law $u = \gamma(\xi) + v$, $v = -k(e)$ and satisfies Assumptions 1–4*, then system* (1)–(3) *is locally asymptotically stable in $\mathbf{W} \times Z \times \Theta \times E$.*

Proof. By using changing supply functions technique [3, 13] for any smooth function $\Delta(z) \geqslant 0$, a controller of form (8) generates $\tau(\mathbf{w})$. This generator with exosystem system $\dot{\mathbf{w}}$ exists in $V(\mathbf{w},z,e)$ and satisfies $\rho_1(\|(\mathbf{w},e)\|) \leqslant V(\mathbf{w},z,e) \leqslant \rho_2(\|(\mathbf{w},e)\|)$ (where $\rho_1(\cdot)$ and $\rho_2(\cdot)$ are some class κ functions) such that the trajectory of the closed-loop system have $\dot{V}(\mathbf{w},z,e) \leqslant -\Delta(z)\|(\chi,e,z,\mathbf{w})\|$.

We assume that the function ϕ is a solution of u^*, $\operatorname{span}\{(u^*(\mathbf{w}), L_s^1 u^*(\mathbf{w}), \ldots, L_s^{d-1} u^*(\mathbf{w}))\} \in \Omega$, and differentiable in $L_s^d u^*(\mathbf{w}) = \phi(u^*(\mathbf{w}), L_s^1 u^*(\mathbf{w}), \ldots, L_s^{d-1} \times u^*(\mathbf{w}))$, suppose that such a case (9) is fulfilled with $\xi = (\xi_0, \ldots, \xi_{d-1})$, $\gamma(\xi) = \xi_0$ such that

$$\tau^{\mathrm{T}}(\mathbf{w}) = \big(\tau_0^{\mathrm{T}}(\mathbf{w}), \ldots, \tau_{d-1}^{\mathrm{T}}(\mathbf{w})\big)^{\mathrm{T}} = \Big(\big(u^*(\mathbf{w})\big)^{\mathrm{T}}, \ldots, \big(L_s^{d-1} u^*(\mathbf{w})\big)^{\mathrm{T}}\Big)^{\mathrm{T}},$$

then it can be solvable by theory of high-gain observers via Lyapunov function technique.

Now consider the positive definite and proper functions

$$V(\mathbf{w},z,e) = V_1(\mathbf{w},z) + \frac{1}{2}e^2 \tag{15}$$

and observe that $V_1(\mathbf{w},z) = \phi(u^*(\mathbf{w}), L_s^1 u^*(\mathbf{w}), \ldots, L_s^{d-1} u^*(\mathbf{w}))$, where the function ϕ is locally Lipschitz and bounded.

Further, by Remark 1, given any smooth function $F(\xi)$ with $\xi = (\xi_0, \dots, \xi_{d-1}) \in \mathbb{R}^{n\times 1}$ attach the internal model (9) to (1) and perform the change of variable $\xi \to \chi$ such that the transforming system (1)–(9) have

$$V_1(\mathbf{w}, z) = \frac{1}{2}\chi^2 + \frac{1}{2}z^2. \tag{16}$$

From [13], we consider the change of variables $\xi \to \overline{\xi} = \xi - \tau(\mathbf{w})$ and apply in equation (1)–(9) with $\gamma(\xi) = \xi_0$. Then

$$\begin{aligned} &\dot{\mathbf{w}} = s(\mathbf{w}), \qquad \dot{z} = f(\mathbf{w}, z, e) \\ &\dot{\overline{\xi}} = A\overline{\xi} + B\overline{\phi}(\mathbf{w}, \overline{\xi}) + Gv, \qquad \dot{e} = q(\mathbf{w}, z, e) + \tau_0(\mathbf{w}) + \overline{\xi_0} + v, \end{aligned}$$

where A is the "shift" matrix and $B = (0, \dots, 0, 1)^{\mathrm{T}}$, $\overline{\phi}(\mathbf{w}, \overline{\xi}) = \phi(\overline{\xi} + \tau(\mathbf{w})) - \phi(\tau(\mathbf{w}))$. Note that $\overline{\phi}(\mathbf{w}, \overline{\xi})$ is locally Lipschitz and bounded for all $\overline{\xi} \in \mathbb{R}^{n\times 1}$ and $\mathbf{w} \in \mathbf{W}$ and $\overline{\phi}(\mathbf{w}, 0) = 0$ for all $\mathbf{w} \in \mathbf{W}$. If $q(\mathbf{w}, z, 0) + \tau_0(\mathbf{w}) = 0$, there exists the change of variable $\overline{\xi} \to \chi = \overline{\xi} - Ge$ with transform system (12) such that

$$\begin{aligned} &\dot{\mathbf{w}} = s(\mathbf{w}), \qquad \dot{z} = f(\mathbf{w}, z, e), \qquad \dot{e} = q(\mathbf{w}, z, e) + \tau_0(\mathbf{w}) + \chi_0 + v, \\ &\dot{\chi} = A\chi + B\overline{\phi}(\mathbf{w}, \chi) - G\big(q(\mathbf{w}, z, 0) + \chi_0 + \tau_0(\mathbf{w})\big) + L(\mathbf{w}, z, \chi, e), \end{aligned}$$

where $L(\mathbf{w}, z, \chi, e) = AGe + B(\overline{\phi}(\mathbf{w}, \chi + Ge) - \overline{\phi}(\mathbf{w}, \chi)) - G(q(\mathbf{w}, z, e) - q(\mathbf{w}, z, 0) + g\lambda_0 e)$.

We assume that a function $q(\mathbf{w}, z, e)$ can be observed as $\overline{q}(\mathbf{w}, z, e) = q(\mathbf{w}, z, e) - q(\mathbf{w}, z, 0)$. If e vanish at $e = 0$, then $\overline{q}(\mathbf{w}, z, e)$ expressed as

$$\begin{aligned} \overline{q}(\mathbf{w}, z, e) &= \int_0^1 \overline{q_{\mathbf{w}}}(\mathbf{w}, z, es)\,\mathrm{d}s \cdot \mathbf{w} + \int_0^1 \overline{q_e}(\mathbf{w}s, z, e)\,\mathrm{d}s \cdot e \\ &= \overline{q_0}(\mathbf{w}, z, e) \cdot \mathbf{w} + \overline{q_1}(\mathbf{w}, z, e) \cdot e. \end{aligned}$$

We assume that the function $q(\mathbf{w}, z, 0)$ can be observed as $\overline{q}(\mathbf{w}, z, 0) = q(\mathbf{w}, z, 0) - q(\mathbf{w}, 0, 0)$, then

$$\overline{q}(\mathbf{w}, z, 0) = \int_0^1 \frac{\partial}{\partial s}\overline{q}(\mathbf{w}s, z, 0)\,\mathrm{d}s = \int_0^1 \overline{q}_{\mathbf{w}1}(\mathbf{w}, z, 0)\,\mathrm{d}s \cdot \mathbf{w} = \overline{q}_{01}(\mathbf{w}, z, 0) \cdot \mathbf{w}.$$

Similarly, we assume that the functions $f_0(\mathbf{w}, z, 0)$, $f(w, z, e)$, and $k(e)$ are expressed as above that satisfy as follows:

$$\begin{aligned} \overline{f}(\mathbf{w}, z, e) &= \overline{f}_0(\mathbf{w}, z, e) \cdot \mathbf{w} + \overline{f}_1(\mathbf{w}, z, e) \cdot e, \\ \overline{f_0}(\mathbf{w}, z, 0) &= f_{01}(0, z, 0) + \overline{f}_{02}(\mathbf{w}, z, 0), \end{aligned}$$

$$\overline{f}_{02}(\mathbf{w},z,0) = \int_0^1 \frac{\partial}{\partial s}\overline{f}_{02}(\mathbf{w}s,z,0)\,\mathrm{d}s = \overline{f}_{22}(\mathbf{w},z,0)\mathbf{w}, k(e)$$
$$= \int_0^1 \frac{\partial}{\partial s}k(es)\cdot e\,\mathrm{d}s \leqslant \overline{k}_1(e)e.$$

Differentiating (16) and substituting (17)–(17) in (16), we have

$$\begin{aligned}\dot{V}_1(\mathbf{w},z) \leqslant \sup_{0\leqslant\chi_0\leqslant\chi}\bigg[&A\chi^2 + BR_1\chi^2 - G\chi^2 + 2\chi^2 - \frac{GM_0}{8}\chi^2 \\ &+ \frac{1}{8}\big((AG + BR_0G - Mg\lambda_0)^2 + N_1^2\big)e^2 \\ &+ \left(\frac{1}{8}(N_2+N_0)^2 + 2\right)z^2\bigg].\end{aligned} \tag{17}$$

Differentiating (15) and substituting (17) gives

$$\begin{aligned}\dot{V}(\mathbf{w},z,e) &\leqslant \sup_{0\leqslant\chi_0\leqslant\chi}\bigg[A\chi^2 + BR_1\chi^2 - G\chi^2 + 3\chi^2 - \frac{GM_0}{8}\chi^2 \\ &\quad + \frac{1}{8}\big((AG + BR_0G - Mg\lambda_0)^2 + N_1^2\big)e^2 + \left(\frac{1}{8}(N_2+N_0)^2 + 2\right)z^2 \\ &\quad + \frac{1}{8}(M_0+M_2)^2\mathbf{w}^2 + (M_1 + K_0 + 3)e^2\bigg] \\ &\leqslant -\Delta(z)\tilde{\eta},\end{aligned}$$

where

$$\Delta(z) = \begin{bmatrix} -A - BR_1 + G - 3 + \frac{GM_0}{8} \\ \frac{-(AG+BR_0G-Mg\lambda_0)^2 - N_1^2}{8} - M_1 - K_0 - 3 \\ -\frac{1}{8}(N_0+N_2)^2 + 2 \\ -\frac{1}{8}(M_0+M_2)^2 \end{bmatrix}, \qquad \tilde{\eta} = \begin{bmatrix}\chi \\ e \\ z \\ \mathbf{w}\end{bmatrix}^2.$$

As in [14], the above inequality shows that $V(\mathbf{w},z,e)$ is bounded, so the solution of the closed-loop system (9) over the control law $u = \gamma(\xi) + v$ is bounded on $[0,\infty)$. By using Barbalat's lemma it concludes that $e(t)$ tends to zero as $t \to \infty$ and system (1)–(3) is locally asymptotically stable in $\mathbf{W} \times Z \times \Theta \times E$. $\square$

Remark 2. The proof of Theorem 1 is infinitesimal version of dissipative passivity inequality for system (1)–(3). To prove global asymptotic stability in $\mathbf{W} \times Z \times \Theta \times E$ we need the exact passivity for (1)–(3), this will be proved via KYP lemma that is the following result.

3 Distribution with passive system

In this section, we are showing how the distribution $\mathbf{w}$ can be passive.

In this work, the general theory summarized in Theorem 1 will be applied to the relevant case of passivity concept. The exosystem $\mathbf{w} = \mathrm{col}(\omega, w)$ can be specified as

$$\dot{\mathbf{w}} = s(\mathbf{w}) = \begin{cases} \dot{\omega} = 0, & \omega \in \Omega_1 \subset \mathbb{R}^{d_1 \times d_1}, \\ \dot{w} = S(w)w, & w \in W \subset \mathbb{R}^{d_2 \times d_2}, \end{cases} \tag{18}$$

where $S(w) = blk\,\mathrm{diag}(S_1(w_1), \ldots, S_r(w_r))$, $S_i = \left(\begin{smallmatrix} 0 & w \\ -w & 0 \end{smallmatrix}\right)$, here r is a set of harmonic oscillators with constant frequencies $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ with the values of ω and w that are unknown but bounded in a compact set $\mathbf{w} \in \mathbf{W} = \Omega_1 \times W \subset \mathbb{R}^{\overline{d}} \times \mathbb{R}^{\overline{d}} \subseteq \mathbb{R}^{d \times d}$, where $\dim d_1 + \dim d_2 = \dim \overline{d}$.

As based on the above situation, the function $u^*(\omega, w)$ introduced as $u^*(\omega, w) = \Gamma(w)\omega, \Gamma(w) = (\Gamma_1(w),\ldots,\Gamma_r(w))$ with $\Gamma_i(w) \in \mathbb{R}^{1\times 2}$, and the pair $(S(w), \Gamma(w))$ is observable for all $\omega \in \Omega_1$ and $w \in W$. This is the case for steady state control input required to enforce zero regulation error, which is a linear combination of r harmonics with uncertain frequencies, amplitudes and phases.

Definition 1. (See [2].) System (1)–(3) with distribution $\mathbf{w}$ is said to passive if there exists a C^1 nonnegative definite storage function V mapping from $V : \mathbb{R}^n \to \mathbb{R}$ with $V(0) = 0$ such that, for all $u \in \mathbb{R}, e \in \mathbb{R}^r$ and $\mathbf{w} \in \mathbf{W}$,

$$V\big(\tau(w)\big) - V\big(\tau(0)\big) \leqslant -\int_0^t \tilde{C}\tilde{S}(w)w\Gamma(w)\omega e^{\mathrm{T}}\,\mathrm{d}\tau, \tag{19}$$

where $\tau(w) = (u^*(w), L_S^1(u^*(w)), \ldots, L_S^{d-1}u^*(w))$ is a solution of (1)–(3).

Assumption 5. Let the matrix S have above expression and is characterized by the polynomial of the block-diagonal if it satisfies the Lie derivative and L_2-norm such that

$$\begin{pmatrix} L_S^0 u^* & \ldots & L_S^{k+2m-1}u^* \\ \vdots & \ddots & \\ L_S^{k+2m+1}u^* & \ldots & L_S^{k+2r}u^* \end{pmatrix} \leqslant \tilde{S}(w)wu^*,$$

where $S = \tilde{S}(w)w$. The next result is KYP property, which described passivity implies global asymptotic stability.

Lemma 1 [Kalman–Yacubovitch–Popov property]. *Suppose that system* (1)–(3) *has a proper C^r, $r \geqslant 1$, in $\mathbf{W} \times Z \times \Theta \times E$. If there exists a Lie derivative $L_{\dot{\mathbf{w}}}V(\tau(w)) - L_{\dot{e}}V(\tau(w)) \leqslant 0$ for all $\tau \in \phi$ and nonnegative storage function V, then system* (1)–(3) *is passive, where*

$$L_{\dot{\mathbf{w}}}V\big(\tau(w)\big) = \frac{\partial}{\partial t}\big(\phi\big(u^*(w), L_s^1 u^*(w), \ldots, L_s^{d-1}u^*(w)\big)\big)\dot{\mathbf{w}},$$
$$L_{\dot{e}}V\big(\tau(w)\big) = \frac{\partial}{\partial t}\big(\phi\big(u^*(w), L_s^1 u^*(w), L_s^{d-1}u^*(w)\big)\big)\dot{e}.$$

Proof. Let $(\partial/\partial w)\phi(u^*(w), L_s^1 u^*(w), \dots, L_s^{d-1}u^*(w)) = \zeta_i(\xi)\phi_i(\xi)$, where $\zeta_m(\xi) = b_m(\xi)$, $\zeta_i(\xi) = b_i(\xi)\prod_{j=i+1}^{m}(1-\zeta_j(\xi))$, $i = 1, \dots, m-1$, and $\zeta : \mathbb{R}^{n\times 1} \to \mathbb{R}^{n\times m}$ is a known function, $\phi_i : \mathbb{R}^{n\times 1} \to \mathbb{R}^{m\times 1}$ is an unknown differentiable function at w, which all are belongs to a compact set $\Sigma \subset \mathbb{R}^{n\times 1}$, where $\Sigma = \{\xi \in \mathbb{R}^{n\times 1}\colon |\phi_i(\xi)| \leqslant n(\xi)$ for all $\xi = \tau(w)$ and $i = 1, \dots, m\}$ and

$$b_i(\xi) = \begin{cases} 1, & \mathbf{w} \text{ is a largest invariant set in } \mathbf{W}, \\ 0, & \mathbf{w} \text{ is not in } \mathbf{W}. \end{cases} \tag{20}$$

Let $\partial\phi/\partial w = \zeta_i(\xi)\phi_i(\xi)$ exists in Σ. Define $\alpha(\xi) = L_{\dot{\mathbf{w}}}V(\tau(w))$. Observe that

$$\max_{\phi_i\in\Sigma}\bigl|\phi_i^{\mathrm{T}}(\xi)\alpha(\xi)\bigr| \leqslant \bigl\|\phi_i(\xi)\alpha(\xi)\bigr\| \leqslant \bigl\|n(\xi)\bigr\|\bigl\|\alpha(\xi)\bigr\|.$$

Now we want to show that there exists indeed a function $\phi_1 = \phi(u^*(w)) \in \Sigma$ such that $\max_{\phi_i\in\Sigma}|\phi_i^{\mathrm{T}}(\xi)\alpha(\xi)| \leqslant \|\phi_1(\xi)\alpha(\xi)\| \leqslant \|n(\xi)\|\|\alpha(\xi)\|$.

Define $|\phi_1^{\mathrm{T}}| = \|n(\xi)\alpha(\xi)\|/\|\alpha(\xi)\|$, whenever $\|\alpha(\xi)\| \neq 0$ and $\phi_1 = 0$. Also whenever $\|\alpha(\xi)\| = 0$ and by construction $\phi_i(\xi) \in \Sigma$, we conclude that $\|\phi_i^{\mathrm{T}}\alpha(\xi)\| = \|n(\xi)\|\|\alpha(\xi)\|$. Similarly let

$$\frac{\partial}{\partial t}\phi\bigl(u^*(w), L_s^1u^*(w), \dots, L_s^{d-1}u^*(w)\bigr) = \frac{\partial\phi_i(\xi)}{\partial w}\frac{\partial w}{\partial e}\frac{\partial e}{\partial t}. \tag{21}$$

Define $\beta(\xi) = L_{\dot{e}}V(\tau(w))$ and

$$\beta(\xi) = \frac{\partial\phi_i(\xi)}{\partial w}\frac{\partial w}{\partial e}\frac{\partial e}{\partial t}\frac{\partial e}{\partial t}, \tag{22}$$

where $\phi_i's$ are unknown functions, which belongs to compact sets $\Lambda \subset \Sigma$ and

$$\Lambda = \biggl\{\mathbf{w} \in \mathbf{W},\ e \in \mathbb{R}^r\colon \frac{\partial w}{\partial e} \leqslant 1 \text{ for all } \xi = \tau(w)\biggr\}.$$

If Theorem 1 holds for all $\phi_i(\xi) \in \Sigma$ and $\xi \in \mathbb{R}^{n\times 1}$, then

$$\beta(\xi) \leqslant -\min_{\phi\in\Sigma}\bigl\{-\phi_i(\xi)^{\mathrm{T}}\beta(\xi)\bigr\} = -\max_{\phi\in\Sigma}\bigl\{\phi_i(\xi)^{\mathrm{T}}\beta(\xi)\bigr\}. \tag{23}$$

By using compactness, equation (22) has $\|\phi_i^{\mathrm{T}}\beta(\xi)\| = \|n(\xi)\|\|\beta(\xi)\|$. Since $\Lambda\cap\Sigma = \{0\}$ for all $\tau(w) \in \Sigma$, the proof is complete. Therefore, we can guarantee that system (1)–(3) is globally asymptotically stable in $\mathbf{W} \times Z \times \Theta \times E$. □

Theorem 3. *Suppose that* $\mathbf{w} = \mathrm{col}(\omega, w) \in \mathbf{W}$, *system* (1)–(3) *has KYP property with* C^1 *in* $\mathbb{R}^{d\times d}\times\mathbb{R}^n\times\mathbb{R}^r$. *If there exists a nonnegative definite storage function* $V : \mathbb{R}^n \to \mathbb{R}$ *with* $V(0) = 0$ *and satisfies the inequality*

$$L_{\mathbf{w}}\bigl(V\bigl(\tau(\mathbf{w})\bigr)\bigr) \leqslant 0, \qquad L_{\dot{e}}\bigl(V\bigl(\tau(\mathbf{w})\bigr)\bigr) \leqslant \widehat{C^*}\bigl[q(w,z,e) + u\bigr]^{\mathrm{T}},$$

where $\widehat{C^*}$ *is a some general constant functions, then system* (1)–(3) *is strictly passive.*

Proof. Under Assumptions 1–5 and Lemma 1, system (1)–(3) has globally asymptotically stable with internal model in $\mathbf{W} \times Z \times \Theta \times E$. Let $\widetilde{V}(\mathbf{w}, z, e)$ be a nonnegative positive definite and proper storage function in C^r ($r \geqslant 1$) for any $\xi_0 = 0$ as $t \to 0$ (where $t \in [0, T] \subset \mathbb{R}$), then by using Lyapunov function for system (1)–(3) with $\widetilde{V}(0, z, 0) = 0$, we get

$$\widetilde{V}(\mathbf{w}, z, e) = \frac{1}{2}\widetilde{V}_1^2\big(\tau(\mathbf{w})\big) + \frac{1}{2}ee^{\mathrm{T}}, \tag{24}$$

where $\widetilde{V}_1(\tau(\mathbf{w}))$ denotes the positive definite and proper function of C^r ($r \geqslant 1$) with distribution $\mathbf{w} = \mathrm{col}(\omega, w)$. Let

$$\dot{\widetilde{V}}_1\big(\tau(\mathbf{w})\big) = L_S^d u^*(w) \quad \forall w \neq 0, \tag{25}$$

and the characteristic polynomial of the block-diagonal matrix S be defined as $P_r(\lambda) = \lambda^{2r} + a_{2r-1}\lambda^{2r-1} + \cdots + a_1\lambda + a_0$ by using Cayley–Hamilton theorem. Then the following equation holds true when $P_r(\lambda) = 0$ for any $(\omega, w) \in \mathbf{W}$ and for any $k \geqslant 0$:

$$\Gamma(w)S^k(w)P_r\big(S^k(w)\big)\omega = 0. \tag{26}$$

If $r > m$, we introduce the coefficients $C_i = a_i/a_{2m}$, $i = 0, \dots, 2r-1$, and $C_{2r} = 1/a_{2m}$, which are well-defined as the coefficient $a_{2m} \neq 0$, and we note that relation (18) implies that

$$\begin{aligned} L_S^{k+2m}u^* &= C_0 L_S^0 u^* + \cdots + C_{2m-1}L_S^{k+2m-1}u^* \\ &\quad + C_{2m+1}L_S^{k+2m} + \cdots + C_{k+2r}L_S^{k+2r}u^* \\ &= \begin{pmatrix} L_S^0 & \dots & L_S^{k+2m-1} \\ \vdots & \ddots & \vdots \\ L_S^{k+2m+1} & \dots & L_S^{k+2r} \end{pmatrix} \times \begin{pmatrix} C_0 \\ \vdots \\ C_{k+2r} \end{pmatrix} u^*. \end{aligned} \tag{27}$$

Taking L_2 norm for (27) and using Assumption 5,

$$\big\|L_S^{k+2m}u^*\big\|_{L_2} \leqslant \tilde{C}\tilde{S}(w)w\Gamma(w)\omega \tag{28}$$

for all $k \geqslant 0$, where $S = S(w)w$, $u^* = \Gamma(w)\omega$, and $\tilde{C}^{\mathrm{T}} = (C_0^{\mathrm{T}}, \dots, C_{k+2r}^{\mathrm{T}})^{\mathrm{T}}$. Substituting (20)–(23) in (24), we have

$$\dot{\widetilde{V}}(\mathbf{w}, z, e) \leqslant \tilde{C}\tilde{S}(w)w\Gamma(w)\omega + \big[(M_0 + M_2)w + (M_1 + K^*)e\big]e^{\mathrm{T}}. \tag{29}$$

If $\min\{(M_0 + M_2), (M_1 + K^*)\} = -\tilde{M}^*$, $\min\{\tilde{C}\tilde{S}(w)\Gamma(w)\} = -\tilde{C}_1\tilde{S}(w)\Gamma(w)$, and $\tilde{V}_1(\tau(w))$ is an available storage function with supply rate $\mathbf{w}$, then based on detectability condition (Lemma 1) $\tilde{V}_1(\tau(w)) \subset \tilde{V}(\mathbf{w}, z, e)$ (see [2]), the third term on the right-hand side of (29) is observed by the second term

$$\begin{aligned} \dot{\widetilde{V}}(\mathbf{w}, z, e) &\leqslant -\tilde{C}_1\tilde{S}(w)w\Gamma(w)\omega - \tilde{M}^* we^{\mathrm{T}} \\ &\leqslant -\big(\big(\tilde{C}_1\tilde{S}(w)w\Gamma(w)\omega\big)^{1/2}\big)^2 - \big(\big(\tilde{M}^* we^{\mathrm{T}}\big)^{1/2}\big)^2. \end{aligned} \tag{30}$$

By using Young's inequality for (30),

$$\widetilde{V}(\mathbf{w}, z, e) - \widetilde{V}(0, z, 0) \geqslant \int\limits_0^t \bigl(\tilde{C}_1 \tilde{S}(w) w \Gamma(w) \omega \tilde{M}^* w e^{\mathrm{T}}\bigr)^{1/2} \,\mathrm{d}s. \tag{31}$$

Taking L_p-norm on both sides for (31) when $p = 2$,

$$\bigl\|\widetilde{V}(\mathbf{w}, z, e) - \widetilde{V}(0, z, 0)\bigr\|_p \geqslant \int\limits_0^t \bigl(\bigl|\tilde{C}_1 \tilde{S}(w) w \Gamma(w) \omega \tilde{M}^* w e^{\mathrm{T}}\bigr|^2\bigr)^{1/2} \,\mathrm{d}s.$$

Let $\tilde{C}_1 \tilde{S}(w) w \Gamma(w) \omega \tilde{M}^* w e^{\mathrm{T}}$ be always positive, and let $1/p + 1/q^1 = 1$ with $p, q^1 > 1$. Then integral Hölder's inequality states that

$$\bigl\|\widetilde{V}(\mathbf{w}, z, e) - \widetilde{V}(0, z, 0)\bigr\|_2 \geqslant \int\limits_0^t \bigl(\bigl|\tilde{C}_1 \tilde{S}(w) w \Gamma(w) \omega\bigr|^2\bigr)^{1/2} \,\mathrm{d}s \int\limits_0^t \bigl(\bigl|\tilde{M}^* w e^{\mathrm{T}}\bigr|^2\bigr)^{1/2} \,\mathrm{d}s.$$

If $\widetilde{C}^* = \max\{\tilde{C}_1, \tilde{M}^*\}$, then $\widetilde{V}(\mathbf{w}, z, e) - \widetilde{V}(0, z, 0) \leqslant -\int_0^t \widetilde{C}^* \tilde{S}(w) w \Gamma(w) \omega w e^{\mathrm{T}} \,\mathrm{d}s$.

Conversely, if system (1)–(3) is passive with C^r, $r \geqslant 1$, the storage function $\widetilde{V}$, taking the derivative with respect to $\mathbf{w}$, t (like the above inequality), clearly implies the theorem statement. Therefore, we can guarantee that system (1)–(3) is globally asymptotically stable in $\mathbb{R}^{d\times d} \times \mathbb{R}^n \times \mathbb{R}^r$. □

Corollary 1. *Suppose that system* (1)–(3) *with disturbances* $\mathbf{w} = \mathrm{col}(\omega, w)$ *is passive. Then system* (1)–(3) *is strictly passive with* C^r*,* $r \geqslant 1$*, for each* $k > 0$ *if there exists a control law* $u = \gamma(\xi) + v$*,* $v = -k(e)$ *such that system* (1)–(3) *is globally asymptotically stabilizable at the equilibrium of* $z = 0$*.*

4 Examples

The equations of motion of aircraft are described by

$$\begin{aligned} &\dot{x}_i = v_i, \qquad \dot{v}_i = g\widehat{e}_3 - \frac{T}{m} R^{\mathrm{T}}(Q_i)\widehat{e}_3, \\ &\dot{Q}_i = \frac{1}{2}\bigl(\eta_i I_2 y + s(q_i) y - q_i^{\mathrm{T}} y\bigr), \qquad \dot{y} = \frac{1}{I_{f_i}}\bigl(\Gamma_i - S(y)\bigr)y, \end{aligned} \tag{32}$$

where $x_i \in \mathbb{R}^3$ and $v_i \in \mathbb{R}^3$ are the position and linear velocity of ith aircraft with respect to the inertial frame $F_0 = (\widehat{e}_1, \widehat{e}_2, \widehat{e}_3)$ and body fixed frame $F_i = (\widehat{e}_{1i}, \widehat{e}_{2i}, \widehat{e}_3)$. Its angular velocity, expressed in $F_i = (\widehat{e}_{1i}, \widehat{e}_{2i}, \widehat{e}_3)$ relative to the fixed fame F_0, is denoted as $y = (y_1, y_2, y_3)^{\mathrm{T}}$. The orientation (attitude) of the ith aircraft is represented using the four-element unit quaternion $Q_i = (q_i^{\mathrm{T}}, \eta_i^{\mathrm{T}})^{\mathrm{T}}$ composed of a vector component $q_i \in \mathbb{R}^3$ and scalar component $\eta_i \in \mathbb{R}$, which are subject to the unity constraint $q_i^{\mathrm{T}} q_i + \eta_i^2 = 1$.

The rotation matrix $\mathbb{R}(Q_i)$ related to the unit quaternion Q_i that brings the initial frame into body frame. I_3 is 3×3 identity matrix, and $S(q_i)$ is the skew-symmetric matrix. The m and g are mass of ith aircraft and gravitation, respectively. The matrix $I_{f_i} \in \mathbb{R}^{3\times 3}$ is the symmetric positive definite constant inertia matrix of the ith aircraft. The scalar T and the vector Γ_i represent the magnitude of the thrust applied to the ith vehicle in the direction of $\widehat{e}_{3i}$ and the external torque applied to the system, which is expressed in F_i.

4.1 Attitude error dynamics

Let the unit quaternion $Q_{d_i} = (q_{d_i}, \eta_{d_i})^{\mathrm{T}}$ represent a desired attitude for the ith aircraft, to be determined later through the control design. The attitude tracking error is described by the discrepancy between the vehicle's attitude and its desired attitude, namely $\widetilde{Q}_i = (\widetilde{q}_i^{\mathrm{T}}, \widetilde{\eta}_i^{\mathrm{T}})$, and is governed by the unit-quaternion dynamics. The angular velocity error vector $\widetilde{y} = y - R(Q_i)y_d$, where y_d is the desired angular velocity of the aircraft, which is related to desired attitude $Q_{d_i} = (q_{d_i}^{\mathrm{T}}, \eta_{d_i}^{\mathrm{T}})$. Then we define

$$\dot{\widetilde{q}}_i = \frac{1}{2}\big(\widetilde{\eta}_i I_3 + S(\widetilde{q}_i)\big)\widetilde{y}_i, \qquad \dot{\widetilde{\eta}}_i = -\frac{1}{2}\widetilde{q}_i^{\mathrm{T}}\widetilde{y}_i\widetilde{y}_i = y - R(\widetilde{Q}_i)y_{d_i},$$

where $y_{d_i} = 2(\eta_{d_i} I_3 + S(q_{d_i}) - q_{d_i}^{\mathrm{T}})\dot{Q}_{d_i}$ and the matrix $R(\widetilde{Q}_i)$ is the rotational matrix related to $\widetilde{Q}_i$, $R(\widetilde{Q}_i) = R(Q_i)R(Q_{d_i})^{\mathrm{T}}$.

With the above assumptions, our objective in this work is to design the UAV aircraft control schemes, in terms of such that the vehicles convergent to a prescribed stationary formulation in the presence of communication signals. Moreover, our objective is guaranteed by $v \to 0$, $|x_i - x_j| < \epsilon$ for each $\epsilon > 0$, $i, j \in N$.

To design a thrust and torque input for the class of under actuated UAVs, equation (32) can be rewritten as

$$\begin{aligned} \dot{x}_i &= v_i, \qquad \dot{v}_i = g\widehat{e}_3 - \frac{T}{m}\big(R^{\mathrm{T}}(Q_i) - R^{\mathrm{T}}(Q_{d_i})\big)\widehat{e}_3, \\ \dot{Q}_i &= \frac{1}{2}\big(\eta_i I_2 y + s(q_i)y - q_i^{\mathrm{T}} y\big), \qquad \dot{y} = \frac{1}{I_{f_i}}\big(\Gamma_i - S(y)\big)y. \end{aligned} \tag{33}$$

The main difficulty in using this extraction algorithm, in this paper, the design of residues can be controlled by $\mathcal{F}_i = g\widehat{e}_3 - (T/m)(R^{\mathrm{T}}(Q_i) - R^{\mathrm{T}}(Q_{d_i}))\widehat{e}_3$ that achieves the formation along with communication signals. Note that the term $\mathcal{F}_i = g\widehat{e}_3 - (T/m)(R^{\mathrm{T}}(Q_i) - R^{\mathrm{T}}(Q_{d_i}))\widehat{e}_3$ can be regulated as a perturbation term that to be translational dynamics in (33).

4.2 Control design reduction (position control design)

To simplify the design of the intermediary translation control and the input torque for each aircraft, which are proposed in this section in two preliminary inputs, that satisfies some of the requirements. Let the auxiliary variable θ associates with state x_i, then

$$\ddot{\theta} = \mathcal{F}_i - u_i, \qquad \gamma = u_i - \phi_i,$$

where $\phi_i \in \mathbb{R}^3$ be the regulated input error signal design, γ be the regulated output design and $u_i \in \mathbb{R}^3$ be additional input vectors to be designed later.

We see that γ is not direct solvable, in this regard, we should introduce some saturation function of *priori* bounds. Therefore, to satisfy the above requirement, we consider the standard formation stabilization control law $\|\mathcal{F}_i\| = \sqrt{3}\sigma_i(k_i^v + \sum_{I,j=1}^{m,n} k_{ij})$.

The function σ_i is a saturation function, that is

$$\sigma_i = \begin{cases} 1, & x_i = x_j \in \mathbb{R}^3, \\ 0, & x_i \neq x_j \in \mathbb{R}^3. \end{cases}$$

The variable k_{ij} is the (i,j)th entry of the weighted adjacency matrix k of the communication signals, which are characterized by the information flow between aircraft.

We propose that an intermediary control input for each aircraft is $\dot{\gamma}_i = -k_{\theta_1}\phi - k_{\theta_2}\dot{\phi}$, where k_{θ_1}, k_{θ_2} are strictly positive scalar gains. By using Therorem 2, the extracted value of the thrust will be used as the real input of the translational dynamics.

We assume that the linear velocity vector is not available for feedback. In other words, we would like to design a linear velocity free global control law that guarantees the boundedness and the asymptotic convergence to zero of the following position and linear velocity tracking errors:

$$e(t) = x_i(t) - x_d(t), \qquad v_i = \dot{e}(t) = v(t) - \dot{x}_d(t). \tag{34}$$

The desired trajectory along the UAV aircraft (32) to be tracked to the allowing thrust and torque.

To design a torque input of (32) without linear velocity measurements, we introduce ξ, where $\xi \in \mathbb{R}^3$ is design variable, which will be determine later. To achieve our objective and solve the above problems, we introduce the following change of variables.

Let $\chi = e - \xi$ be an change variable, and also it will be new error signals, which may depends on explicitly of the linear velocity of the aircraft, that is $z = \dot{\chi} = \dot{e} - \dot{\xi} = v_i - \dot{\xi}$. Then to design the attitude tracking torque, we introduce the following variable:

$$\varOmega_* = \widetilde{y} - \beta, \tag{35}$$

where $\widetilde{y} = y - R(\widetilde{Q})y_d$ is angular velocity error vector, y_d is the desired angular velocity of the aircraft, and β is a design parameter. Exploiting the rotational dynamics in (32) and expression (34), we can easily show that $\dot{\varOmega}_* = \dot{\widetilde{y}} - \dot{\beta}$. It is clear that $\widetilde{y} = y - y_d$, which is interlink with new error signals $\widetilde{z} = z - \widehat{z}$, this is depends on the explicit linear velocity of the aircraft.

We define the following input torque for each aircraft: $\varGamma_i = I_{f_i}\dot{y} + I_{f_i}S(y_d)\dot{y}_d\dot{y} - k_{q_1}\dot{\beta} - k_{q_2}\widetilde{q}$. Therefore,

$$I_{f_i}\dot{\varOmega}_* = \tau - S(y)I_{f_i}y - I_{f_i}S(\widetilde{y})R(\widetilde{Q_i})y_d + I_{f_i}R(\widetilde{y})\gamma - \dot{\beta})I_{f_i} - \varGamma_i, \tag{36}$$

where $\dot{y} - \dot{y}_d = \tau - S(y)I_{f_i}y - I_{f_i}S(\widetilde{y})R(\widetilde{Q_i})y_d + I_{f_i}R(\widetilde{Q_i})\gamma$.

To design the input torque for the rotational dynamics, we consider the extracted value of the desired attitude Q_{d_i}, which is act as time-varying reference attitude, that is

$$y_d = S(y)\dot{\gamma}, \qquad \dot{y}_d = S(y)\dot{y}\dot{\gamma} + S(y)\ddot{\gamma}, \tag{37}$$

where $\dot{\gamma}, \ddot{\gamma}$ are first and second derivative of γ.

Substitute (37) in (36), we get

$$I_{f_i}\dot{y}_d = \big(\dot{y} - S(y)\dot{y}\dot{\gamma} + S(y)\ddot{\gamma} - \dot{\beta}\big)I_{f_i} - \Gamma_i. \tag{38}$$

To design a torque input in (37) without linear-velocity measurements, we introduce the following nonlinear observer, that is $\widetilde{z} = \dot{\xi} = v_i - L_1\widetilde{\xi}$, $\dot{v}_i = -k_{\theta_1}\phi - k_{\theta_2}\dot{\phi} + \Gamma_i^{\mathrm{T}}\Omega_* - L_1\widetilde{\xi}$, where $\widetilde{z} = z - \widehat{z}$ generates estimates of the linear velocity vector $\widehat{z}$, and $k_{\theta_1}, k_{\theta_2}L_1$ are positive gains, $\widetilde{\xi} = \xi - \widehat{\xi}$ is error observer, and $\widehat{\xi}$ is error desired.

The error vector is defined as

$$\widetilde{z} = \dot{\widetilde{\xi}} = z - \widehat{z}, \qquad \dot{\widetilde{z}} = k_{\theta_1}\phi + k_{\theta_2}\dot{\phi} + L_1\widetilde{z} + \Gamma_i\Omega_* - L_1\widetilde{\xi}. \tag{39}$$

To achieve our objective, that is prove the passivity, we need to consider the following definite functions $V_1 = (1/2)(\widetilde{z}\widetilde{z}^{\mathrm{T}} + k_{t_1}\chi^{\mathrm{T}}\chi + k_{t_2}(\chi - \gamma)(\chi - \gamma)^{\mathrm{T}})$, where $(\chi - \gamma)$ is output error of the auxiliary systems, which plays the role of estimation of control design at the stage of linear velocity, and k_{t_1}, k_{t_2} are positive constants.

Therefore,

$$\begin{aligned}\dot{V}_1 &= \widetilde{z}\big[-\dot{v}_i - k_{\theta_1}\phi - k_{\theta_2}\dot{\phi} - \big(I_{f_i}\dot{y}_d + I_{f_i}S(y)\dot{y}\dot{\gamma} - k_{q_1}\dot{\beta} - k_{q_2}\widetilde{q}\big)(\widetilde{y} - \dot{\beta}) - L_1\widetilde{\xi}\big] \\ &\quad + k_{t_1}\chi^{\mathrm{T}}(v_i - \dot{\xi}) + k_{t_2}(\chi - \gamma)^{\mathrm{T}}(v_i - \dot{\xi} + k_{\theta_1}\phi + k_{\theta_2}\dot{\phi}).\end{aligned} \tag{40}$$

To derive the torque input stability, we consider the following positive definite function:

$$V_2 = \frac{1}{2}(\widetilde{z} + \widetilde{\xi})^{\mathrm{T}}(\widetilde{z} + \widetilde{\xi}) + \frac{1}{2}L_{v_2}\widetilde{\xi}\widetilde{\xi}^{\mathrm{T}} + 2k_{v_2}(1 - \widetilde{\eta}) + \frac{1}{2}I_{f_i}\Omega_*^{\mathrm{T}}\Omega_*, \tag{41}$$

where L_{v_2}, k_{v_2} are positive constants.

In view of the above equation, we propose the torque input for rotational dynamics

$$\tau = S(y)I_f y - I_f S(\widetilde{y})R(\widetilde{Q}_i)y + I_f\dot{\beta} - k_{q_1}\widetilde{q} - k_{\Omega_*}\Omega_* - \Gamma_i(\widetilde{z} + \widetilde{\xi}). \tag{42}$$

Differentiate (41) w.r.t. t and substitute in (35), we get

$$\begin{aligned}\dot{V}_2 &= (\widetilde{z} + \widetilde{\xi})^{\mathrm{T}}(\dot{v}_i + k_{\theta_1}\phi + k_{\theta_2}\dot{\phi} + \Gamma_i^{\mathrm{T}}\Omega_* - L_1\widetilde{\xi} + z - \widetilde{z}) + L_{v_2}\widetilde{\xi}^{\mathrm{T}}(z - \widetilde{z}) \\ &\quad + k_{v_2}(1 - \dot{\widetilde{\eta}}) + (\widetilde{y} - \beta)^{\mathrm{T}}\big(\tau - S(y)I_f y + I_f S(\widetilde{y})R(\widetilde{Q}_i)y_d - I_f\dot{\beta} + k_{q_1}\widetilde{q} \\ &\quad + k_{\Omega_*}\Omega_* + \Gamma_i(\widetilde{z} + \widetilde{\xi})\big).\end{aligned} \tag{43}$$

4.3 Passivity of the overall system

We check that desired trajectory and the controller gains of the UAV vehicle, which is possible to extract the magnitude of the thrust and torque of the desired attitude. In particular, control design has been fixed, at this stage, all the required signals will be observable and well-defined. In this regard, consider the Lyapunov function $V = V_1 + V_2$. Therefore,

$$\dot{V} = \dot{V}_1 + \dot{V}_2. \tag{44}$$

Substitute (40) and (43) in (44), we get

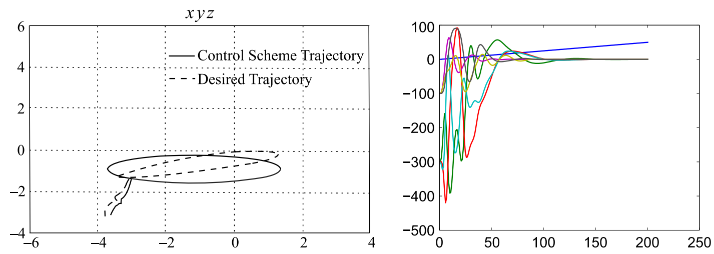
$$\begin{aligned}\dot{V} &= \widetilde{z}\big[-\dot{v}_i - k_{\theta_1}\phi - k_{\theta_2}\dot{\phi} - \big(I_{f_i}\dot{y}_d + I_{f_i}S(y)\dot{y}_d\dot{\gamma} - k_{q_1}\dot{\beta} - k_{q_2}\widetilde{q}\big)(\widetilde{y} - \dot{\beta}) - L_1\widetilde{\xi}\big] \\ &\quad + k_{t_1}\chi^{\mathrm{T}}(v_i - \dot{\xi}) + k_{t_2}(\chi - \gamma)^{\mathrm{T}}(v_i - \dot{\xi} + k_{\theta_1}\phi + k_{\theta_2}\dot{\phi}) \\ &\quad + (\widetilde{z} + k_{\theta_1}\phi + k_{\theta_2}\dot{\phi} + \mathit{\Gamma}_i^{\mathrm{T}}\mathit{\Omega} - L_1\widetilde{\xi} + \dot{\widetilde{\xi}})^{\mathrm{T}}(\dot{v}_i + z - \widetilde{z}) \\ &\quad + L_{v_2}\widetilde{\xi}^{\mathrm{T}}(z - \widetilde{z}) + k_{v_2}(1 - \dot{\widetilde{\eta}}) + (\widetilde{y} - \beta)^{\mathrm{T}}\big(\tau - S(y)\big)I_f y \\ &\quad + I_f S(\widetilde{y})R(\widetilde{Q})y_d - I_f\dot{\beta} + k_{q_1}\widetilde{q} + k_{\mathit{\Omega}_*}\mathit{\Omega}_* + \mathit{\Gamma}_i(\widetilde{z} + \dot{\widetilde{\xi}}).\end{aligned}$$

If $S(y) = S(\widetilde{y})$, $\|R(\widetilde{Q})\| \leqslant 1$ and using $2ab \leqslant \epsilon a^2 + b^2/\epsilon$,

$$\begin{aligned}\dot{V} &\leqslant -(k_{t_1} - 2)\|v\|^2 - k_{t_2}\|\dot{\xi}\| - \left(\frac{3}{4} - k_{\theta_1} - k_{\theta_1}\right)\big\|\psi^{\mathrm{T}}\big\|^2 - (k_{\theta_1} + k_{\theta_1})\big\|\gamma^{\mathrm{T}}\big\|^2 \\ &\quad - (-k_{\theta_1} - k_{\theta_1})\|\phi\|^2 - (k_{\theta_1} + k_{\theta_1})\|\dot{\phi}\|^2 - \frac{1}{4}\|\dot{\xi}\|^2 - (1 - L_{v_2})\|\dot{\xi}^{\mathrm{T}}\|^2 \\ &\quad - \|\mathit{\Gamma}_i\|^2 - \frac{3}{4}\|\mathit{\Omega}_*\|^2 - \frac{L_{v_2}}{4}\|\widetilde{z}\|^2 + k_{v_2}(1 - \dot{\eta}) - \|\dot{\widetilde{y}}\|.\end{aligned}$$

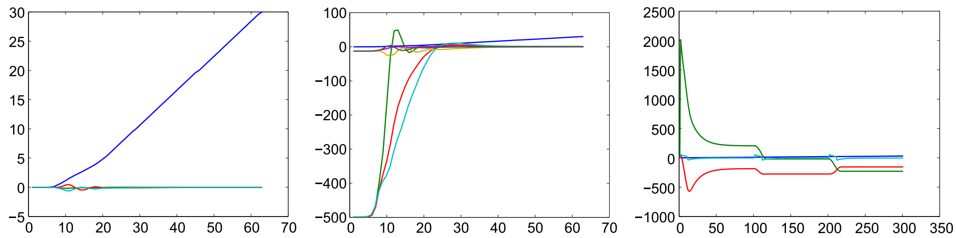
Therefore, $\dot{V}$ is negative semi-definite, we can conclude that $V, \xi, \phi, \gamma, \xi, \mathit{\Gamma}_i, \mathit{\Omega}_*, \widetilde{z}, \widetilde{y}$ are bounded. Consequently, by using Barbalat's lemma, we can conclude that $(\chi - \gamma) \to 0$, $\widetilde{z} \to 0$, and $q \to 0$. Since $\widetilde{z}$ and z converges to zero, it is clear that $\widehat{z}$ tends to zero. If $\mathit{\Gamma}_i \to 0$, which implies that $\dot{\beta} \to 0$, then $e \to 0$ and $v \to 0$ asymptotically.

To complete the proof, we must show that the thrust input T is bounded. Exploiting the above boundedness, we can show that if $\gamma \to 0$, then it is bounded. By taking derivative of (42), we can show that $\dot{T} = -\dot{\tau}$. If τ is bounded, which implies that $\dot{T}$ is bounded. Using the above boundedness results, it is clear that T and y_d and y are bounded. So we conclude that $Q_i(t)$ is bounded and $\lim_{t\to\infty} Q_i(t) = x_i(t)$. In Fig. 1, the UAV of entire output regulation with feedback control scheme of the desired trajectory is illustrated. This output feedback control scheme consists of a kinematic control, to generate the desired velocity of the dynamic controller and virtual controller, which provides the quaternion Q_i that are observed by the estimate of the states. Therefore, system (32) is passive. As based on Theorem 3.1 (see [6]), passivity is guaranteeing the global asymptotic stability of (32).



(a) (b)

Figure 2. (a) 3D plot of the UAV vehicle trajectory with desired trajectory response; (b) The behavior of state x (solid line) with desired velocity x_d (dot line) response with disturbance $w(t)$.



(a) (b) (c)

Figure 3. (a) The behavior of velocity response; (b) The behavior of attitude $\widetilde{q}$ (solid line) with desired attitude $\widetilde{q}_d$ (dot line) response; (c) The behavior of angular velocity y (solid line) and desired angular y_d velocity (dot line) response.

4.4 Simulation results

Simulations results are presented to illustrate the effects of proposed control scheme. In this scheme, we consider some basic parameters: $v(0) = (0, 0, 0)$, $g = 9.8$, $k_{t_1} = 0.1$, $k_{t_2} = 0.2$, $k_{\theta_1} = k_{\theta_2} = 0.5$, $m = 3kg$, and $I_{f_i} = \operatorname{col}(0.1, 0.1, 0.1)$. Among three control models, the position stabilization is most advanced one and the simulations are only presented for this model. Theorem 2 is applied for equation (32), the desired yaw angular velocity $y_d = 0$ is set to zero. In [11], the helicopters model with four control parameters like as track vertical position, lateral, longitudinal, and yaw attitude time reference $z_r(t)$, $y_r(t)$, $x_r(t)$, and $\psi_r(t)$ were used, attitude with engine dynamics are independent. But the engine dynamics are not only related to attitude, it is also related to control force and communication signals. In our paper, vehicles attitude and angular velocity are related to quaternion term Q_{d_i} and control force $\mathcal{F}_i$. Comparing [11], our result Fig. 2 shows clear performance of take up and landing. In [11], the mass M and inertia matrix J was defined as 8 and $\operatorname{diag}(0.18, 0.34, 0.28)$ kg m^2, and PID is maintained as constant value like as $K_P = 30, K_I = 0.007, K_D = 0.185$, respectively. In our paper, mass m and inertia matrix I is fixed as 3 and $\operatorname{diag}(0.1, 0.1, 0.1, 0.1)$ kg m^2, respectively, and no fixed PID is used.

The disturbance is a fast ramp to simulate a gust of wind with MATLAB Simulink, which is applied to the position in 3-D. Figure 2(a) demonstrates the UAV ability to follow a trajectory in 3-D, while in presence and absence of unmatched uncertainties. The unmatched uncertainty are considered in $S(y)$. The obtained results in this case are given in Figs. 2(b) and 3(a), which illustrate the aircraft linear velocities and positions in spaces, respectively. We can see from Figs. 2(b) and 3(a) that our control objective is achieved in the presence of uncertainties, whose system whereas both cases convergence to almost the same steady-state conditions. Figure 3(b) shows the attitude tracking error, and Fig. 3(c) illustrates the desired and actual angular velocity of the aircraft. It is clear from these figures that asymptotic convergence to zero is guaranteeing the stability.

5 Conclusions

In this theoretical analysis, we investigated the problem of exosystem signals with adaptive nonlinear system of factorable low-high frequency gains. The necessary conditions are derived via minimum-phase system and KYP property, to grant the passivity. The passivity is granting the globally asymptotic stability. In particular, the new method has been developed in a general framework handling the case of over-dimension, under-dimension with adaptive output regulation of minimum phase system with passivity. This result incorporates and extends a number of stabilization schemes and expressly applicability of theory result demonstrated in aircraft motions. Moreover, compared with differential method [2, 11], the proposed system with passivity is able to achieve a lower attention level. This is shown in example with simulation results even in the special case where the desired controlled system consist the passivity.

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