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DYNAMICS OF THE PROCESSES IN METAL MACHINING

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In the practice of processing of metals by cutting it is necessary to overcome the vibration of the cutting tool, the processed detail and units of the machine tool. These vibrations in many cases are an obstacle to increase the productivity and quality of treatment of details on metal-cutting machine tools. Vibration at cutting of metals is a very diverse phenomenon due to both it's nature and the form of oscillatory motion.

The most general classification of vibrations at cutting is a division them into forced vibration and autovibrations. The most difficult to remove and poorly investigated are the autovibrations, i.e. vibrations arising at the absence of external periodic forces. The autovibrations, stipulated by the process of cutting on metal-cutting machine are of two types: the low-frequency autovibrations and high-frequency autovibrations. When the low-frequency autovibration there appear, the cutting process ought to be terminated and the cause of the vibrations eliminated. Otherwise, there is a danger of a break of both machine and tool. In the case of high-frequency vibration the machine operates apparently quiently, but the processed surface feature small-sized roughness. The frequency of autovibrations can reach 5000 Hz and more.

1. DYNAMICS OF CUTTING PROCESS

1.1. Statement of the problem. The stability of process of removal of a shavings in a wide range of technological modes is one of the main requirements for metal-cutting machine tool. In order to build a theory of autovibration at cutting, it is necessary to consider the laws of deformation of metal under the process. Peculiarity of process of cutting related to plasticity properties of metal causes delay in change of field of stresses and, consequently, delay of forces, acting on lathe tool with respect to coordinates of the latter. The autovibration at cutting of metal is generated by the delay forces that shake the system. The conditions of stability of such system can be established by usual methods of analysis of linearised equations, describing vibration.

1.2. Delay of forces at cutting of metals. The reason of delay of forces at cutting of metal is the peculiar process of deformation of metal. For this purpose we shall consider conditions of formation of shavings. At removal of shavings C (Fig.1) from either steel or cast iron an advancing crack *afg* at the edge a of lathe tool is observed. At cutting of elastic metals having properties of plasticity, hardening and fragility, the periodic occurrence of a crack *afg* is inevitable. To begin with, we assume that deformation of the layer *abdf* (Fig.2) is in the initial stage and there is no crack yet.

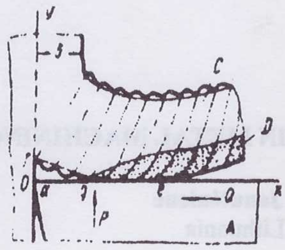

Fig. 1.

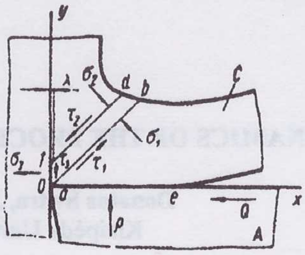

Fig. 2.

Assume that the inclined surfaces *ab* and *fd*, bounding the layer, coincide with the surfaces of the heaviest tangential stress. Then it is possible to consider a layer *abdf* as if it were in balance under the action of the following stresses: normal σ_1, σ_2 and tangential τ_1, τ_2 acting on the layer along the surfaces *ab* and *fd*, as well as the normal stress σ_3 and stress of shift τ_3, acting on the surface *af* separating a deformable layer from the processed metal. It is obvious, that deformations of crushing and shift of layer *abdf* in directions σ_1, σ_2, τ_1, and τ_2 are accompanied with deformation of shift in *y*-plane. Assuming that the edge of lathe *a* moves in *y*-plane, i.e. the thickness of layer λ exposed to deformations of shift is close to zero, it is obvious, that even a small movement of lathe in *y*-plane induce significant relative shift. Therefore tangential stress τ_3 in the points of *a* quickly reaches both yield and strength limit. Hence in these points a crack will appear. At further movement of lathe the displacement of points in *af*-plane increase under the compression of layer *abdf*, therefore causing a spread of the crack up to point *f*. The high concentration of stress in top of crack prootes increase of rate of its formation. Thus deforming element will be cut off to the length *af* and stress σ_3 will vanish, as length *af* will reach some critical size. As a consequence of change of equilibrium condition of element *abdf*, the resistance of the element to displacement with respect to *fd* decreases so, that compressive deformation of the element pressed before stops and total replacement of shavings will be carried out at the expense of this shift at simultaneous local crushing in corner *fag* (Fig.1). An advancing crack turns to plane *afg*. At further deformation due to approach of an edge *a* of lathe to vertice *f* of the cavity, the latter shrinks down up to the close contact of the edge with the points *f*. The process of reduction of cavity is accompanied by both increase of resistance and resumption of deformation of crushing of cutting layer, that is the beginning of formation of the next element.

The described discontinuous process of formation of the shavings has a rather high frequency and is accompanied by corresponding oscilation of cutting force. The edge of lathe *a* does not operate continously removing the main shavings, but only incise layer of the material.

Thus, at small vibrations of the system in direction of *x* the oscilation of thickness of the shavings and force *P* is periodically detained and it occurs intermittently. Time of delay of cutting force *P*, as well as the force itself, depends on *x*. If to disturb the system in the direction of *x*, then due to delay the lathe will pass some way l_r in direction of y. Denote as h_p the time delay, which is necessary to run the delay distance l_p. The delay equation relating force P and coordinate has a form

$$-\Delta P\,(t) = B\Delta x\,(t - h_p) \tag{1.1}$$

where $\Delta P(t)$ – function of t, $\Delta x(t - h_p)$ function of $(t - h_p)$.

Further consideration of process of deformation of metal at cutting results in conclusion [8], that at small fluctuations of system, delay of friction force Q with respect to force P should take place. At the surface of contact of shavings C with the lathe in zone D (Fig.1) there is the second plastic deformation. The zone D is formed under the action of friction force Q, which tends to move metal on the sliding surface in the direction opposite to the movement. At presence of outrun cavity the normal and tangential stresses, that are equal to zero on surface fg, grow on region ge. As force P receives increment ΔP, the normal stress σ_y on surface ge increases. Obviously, tangential stress τ_y on this surface will increase only after the metal at lathe will receive some additional shift.

Therefore, each element of shavings being moved along the surface of lathe causes the variable elementary friction force that reaches its maximal value only after the element passes the way l_q. As the cutting force receives increment ΔP, increment ΔQ of friction force Q reaches the size $\Delta Q = f\Delta P$ only through some time h_Q, which is necessary to move shavings along the way l_Q. Similarly to equation (1.1), equation of delay of friction force has a form

$$\Delta Q\,(t) = f\Delta P\,(t - h_Q). \tag{1.2}$$

The linear analysis of difference-differential equations (1.3) – (1.4) is provided in [8]. Our objective is the complete linear and nonlinear analysis of system (1.1) – (1.2) applying methods of theory of bifurcation, advanced in [9].

Dynamic equations in the case of small oscillations we put into the following form:

$$\ddot{x}(t) + \frac{D(x)}{m_x} + w_y^2 x(t) = \frac{Q(t)}{m_x} \qquad \left(w_x^2 = \frac{c_x}{m_x}\right) \tag{1.3}$$

$$\ddot{y}(t) + \frac{D(y)}{m_y} + w_y^2 y(t) = \frac{P(t)}{m_y} \qquad \left(w_y^2 = \frac{c_y}{m_y}\right) \tag{1.4}$$

where $D(x)$ and $D(y)$ – dissipative forces, m_x and m_y – masses, c_x and c_y – coefficients of elasticity. Time of delay h_p depends on $v_S + \Delta\dot{y}$, time of delay h_Q depends on $v_S + \Delta\dot{y} / \xi_O + \Delta\dot{x}$, i.e. h_p and h_Q are variable.

2. DYNAMICS OF DRILLING PROCESS

2.1. Introduction. Autovibration arising in the process of drilling on drilling machine, result in a series of harmful phenomena: decrease of accuracy and quality of processing, decline of productivity of the machine tool, its anticipatory wear, decrease of resistance of the tool, and break of drill or boring bar at deep drilling.

2.2. Origin of excitation and equations of motion. The principle scheme of drilling machine is represented on Fig 3. There are two distinct oscillatory systems in a drilling machine: firstly, a spindle with gears of the drive of rotation (oscillator φ), secondly, the whole spindle unit (oscillator s).

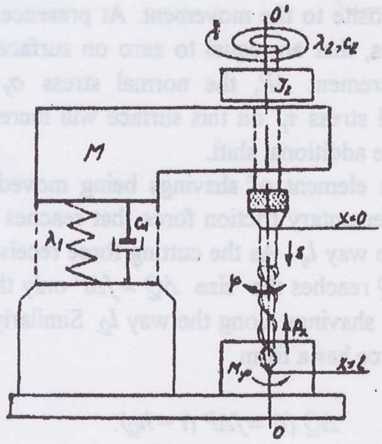

Fig. 3.

Assuming that drill is absolutely stiff, we shall show the origin of excitation that takes place at drilling. In process of drilling, change of feed s and velocity of feed $\dot{s}_0$ results in change of magnitude of torque M_φ, while the change of angular velocity of drill $\dot{\varphi}$ changes axial cutting force P_x. The possible reason of inhibition of vibration are the delay forces whose delay is equal to the duration of rotation of the drill by m-th part of the full angle, if there is m cutting edges on the drill. In the case of deep drilling, one can not consider a drill to be absolutely stiff, therefore the delay, that is equal to time of run of an elastic wave along the whole length of a drill, plays the main role in inhibition of vibration.

We shall consider the detail as unmovable, the motor rotating at constant angular speed Ω and the drill in direction of feed to be absolutely stiff.

In the case of nonvibrating mode of drilling, both axial component of cutting force P_x and torque M_φ are the functions of two independent variables, only, i.e. cutting velocity $v_0 = \Omega_r$ (here r is a radius of drill) and velocity of feed s_0. In this case velocity of feed s_0 is related to v_0 and s_0 by the following equation:

$$\dot{s}_0 = s_0 \Omega / 2\pi .$$

In the case of vibrating mode of drilling, the increment of feed velocity can vary independently to both the increment of cutting velocity $v = r\dot{\varphi}|_{x=l}$ and feed s; therefore the following hold:

$$P_x = P_x(s_o + \Delta s,\ \dot{s}_o + \dot{s},\ v_0 + v),$$
$$M_\varphi = M_\varphi(s_o + \Delta s,\ \dot{s}_o + \dot{s},\ v_0 + v). \qquad (2.1)$$

Here Δs – increment of thickness of shavings, $\varphi\ (x,t)_{x=l}$ – increment of angular velocity of rotation of drill end. The thickness of shavings, cut by each edge at a given moment of time, depends on trace on surface, formed by previous edge h seconds earlier (fig.4), therefore

$$\Delta S = m\ [s(t) - s(t-h)], \text{ where } h = 2\pi r / m(v_0 + v)\big|_{x=l}\ . \qquad (2.2)$$

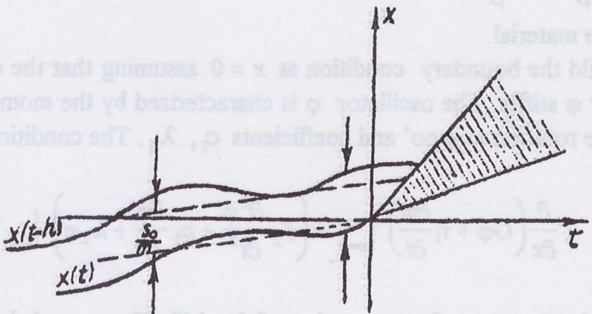

Fig.4. Change of thickness of drill shavings during $1/m$ of a turn of the drill.

In the case of vibrating mode of drilling, the increments of cutting forces in the neighbourhood of s_0, $\dot{s}_0$, v_0 are equal

$$dP_x = k\ \Delta s + k\dot{s} + k\ v(l,\ t),$$

$$dM_\varphi = n\ \Delta s + n\dot{s} + n\ v(l,\ t), \qquad (2.3)$$

where

$$k_1 = \left(\frac{\partial P_x}{\partial x}\right)_0,\quad k_2 = \left(\frac{\partial P_x}{\partial \dot{s}}\right)_0,\quad k_3 = \left(\frac{\partial P_x}{\partial v}\right)_0,$$

$$n_1 = \left(\frac{\partial M_\varphi}{\partial s}\right)_0,\quad n_2 = \left(\frac{\partial M_\varphi}{\partial \dot{s}}\right)_0,\quad n_3 = \left(\frac{\partial M_\varphi}{\partial v}\right)_0$$

The equation of oscillation of oscillator s at drilling have a form

$$M_1\ddot{s} + c_1\dot{s} + \lambda_1 s = -dP_x\ , \qquad (2.4)$$

where M_1 is the mass of the whole spindle unit of the machine tool, concentrated at $x = 1$, c_1, λ_1 – coefficients of resistivity and elasticity forces.

In the direction of turning the drill is considered as an elastic core, stiffly attached to a chuck of a spindle.

Let $\varphi(x,t)$ be an increment of an angle of a turn of cross section of the drill located on distance x from the attachment point. This imply the following equation of rotational vibration of a drill:

$$\frac{\partial^2 \varphi}{\partial t^2} = a\frac{\partial^2 \varphi}{\partial x^2} + b\frac{\partial^3 \varphi}{\partial x^2 \partial t}, \tag{2.5}$$

where $a = \frac{G}{\rho}$; $b = \frac{\eta}{\rho}$, η – coefficient of internal friction, G – magnitude of shift, ρ – density of the material.

We build the boundary condition at $x = 0$ assuming that the drill is attached to the oscillator φ stiffly. The oscillator φ is characterized by the moment of inertia with respect to the rotation axis oo' and coefficients c_2, λ_2. The condition has a form:

$$I\frac{\partial}{\partial x}\left(G\varphi + \eta\frac{\partial \varphi}{\partial t}\right)\Big|_{x=0} = \left(I_2\frac{\partial^2 \varphi}{\partial t^2} + c_2\frac{\partial \varphi}{\partial t} + \lambda_2 \varphi\right)\Big|_{x=0} \tag{2.6}$$

where I – polar moment of cross section of the drill. The second boundary condition at $x = 1$ is obtained from equality of moments on the end of drill at cutting:

$$I\frac{\partial}{\partial x}\left(G\varphi + \eta\frac{\partial \varphi}{\partial t}\right)\Big|_{x=l} = -dM_{\varphi}\Big|_{x=l} \tag{2.7}$$

Differentiating the equations (2.5), (2.6), (2.7) with respect to t and substituting $\dot{\varphi} = \frac{v}{r}$ we come to equation for v:

$$\frac{\partial^2 v}{\partial t^2} = a\frac{\partial^2 v}{\partial x^2} + b\frac{\partial^3 v}{\partial x^2 \partial t} \tag{2.8}$$

with the boundary conditions:

$$\frac{I}{\lambda_2}\left(G\frac{\partial v}{\partial x} + \eta\frac{\partial v^2}{\partial x \partial t}\right)\Big|_{x=0} = \left(\frac{1}{\omega_2^2}\frac{\partial^2 v}{\partial t^2} + \frac{c_2}{\lambda_2}\frac{\partial v}{\partial t} + v\right)\Big|_{x=0} \tag{2.9}$$

$$\frac{1}{\omega_2^2}\frac{\partial^2 s}{\partial t^2} + \delta_1\frac{sv}{\partial t} + s + \frac{k_1 m}{\lambda_1}\left[s(t) - s(t-h) + \frac{k_3}{\lambda_1}v(l,t)\right] = 0 \tag{2.10}$$

$$\frac{I}{r}\left(G\frac{\partial v}{\partial x}+\eta\frac{\partial v^2}{\partial x\partial t}\right)\Big|_{x=l}=\frac{d}{dt}\left(n_1[s(t)-s(t-h)+n_2\frac{ds}{dt}+n_3 v\right)\Big|_{x=l} \quad (2.11)$$

where ω_1, and ω_2 – eigenfrequencies of the oscillators s and $\varphi(0,t)$; $\delta_1=(c_1+k_2)/\lambda_1$, $h=2\pi r/v_0$. The linear analysis of model (2.8)–(2.11) is conducted in [6].

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