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Finite-time stabilization for fractional-order inertial neural networks with time-varying delays

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Abstract. This paper deals with the finite-time stabilization of fractional-order inertial neural network with varying time-delays (FOINNs). Firstly, by correctly selected variable substitution, the system is transformed into a first-order fractional differential equation. Secondly, by building Lyapunov functionalities and using analytical techniques, as well as new control algorithms (which include the delay-dependent and delay-free controller), novel and effective criteria are established to attain the finite-time stabilization of the addressed system. Finally, two examples are used to illustrate the effectiveness and feasibility of the obtained results.

Keywords: inertial neural networks, finite-time stabilization, fractional-order system, Caputo fractional derivative and integral.

1 Introduction

In the last decade, dynamic analysis has developed in many disciplines such as economic sciences, ecology and environment, biology and engineer, etc. [1–4]. Especially, the artificial neural networks (ANNs) dynamical systems are applied in particular to solve problems of classification, prediction, categorisation, optimization, recognition of forms, associative memory secure communication [6–8]. In recent years, more outcomes on equilibrium-point stability, bifurcation, periodicity analysis and synchronization of various types of recurrent networks has been widely investigated in [9, 10]. In 1986, Babcock and Westervelt introduced the inertial neural network, which was characterized by the second-order differential equation [11]. The addition of inertial terms in electronic neural networks may give rise to the complicated behaviors such as instability, spontaneous concussion and chaotic behavior. Recently, integer-order inertial neural networks have attracted considerable attention, and numerous excellent results have been published [5, 12, 13]. The finite-time and fixed-time synchronization of a class of inertial neural networks with multiproportional delays were obtained using Lyapunov functionals and analytical techniques [5]. In [12], the problem of synchronization and periodicity of coupled inertial memristive neural networks with supremums were studied by using the matrix measure method and Halany inequality techniques. Sufficient conditions on global asymptotic synchronization of inertial delayed neural networks by using integrating inequality techniques was explored [13].

As an extension of integer integral and derivative to arbitrary order, fractional calculus is a field of pure mathematical theory, whose purpose is to extend the definitions of traditional integrals and derivatives to noninteger orders. It remains an open problem in signal processing, laser physics, secure communication, automatic, electricity, electrochemistry and in many search fields due to the nature of these systems, which are considered as long memory systems, and they present a complex dynamic [14]. Although, the mathematical formalism of the noninteger derivative associated with the development of computer tools allowed to envisage applications in the field of science of the fractional-order differential engineer. Today, the fractional approach is applied to the modelling of the consequences of natural disasters [15], to the modelling of electrical devices [16] or to the synthesis of the control [17]. Fractional modelling is also present in the field of the humanities and social sciences or even in biological sciences [18]. Due to the extensive applications, some interesting and important results on fractional-order neural networks have been obtained [19].

In recent studies, the finite-time stability with control of neural networks have been intensively considered within the system solution reach the equilibrium point in finite time. The time function is called the time convergence or the settling time. The finite-time stability has a greater importance than the usual asymptotic stability in real applications like robot, optimization problems, pattern recognition, vehicle system and identification and spacecrafts of dynamical systems [20]. Stabilization control of neural networks has attracted more and more attention, and there are different types of controllers such that pinning adaptive control [21], intermittent control [22], fuzzy control [23], sliding mode control [24]. In [25], the finite-time Mittag-Leffler synchronization of fractional-order memristive BAM neural networks with time delays was investigated based on Lyapunov theory and linear feedback controller. The graph theory-based finite-time synchronization of fractional-order complex dynamical networks was analyzed based on analysis techniques and algebraic graph theory method [27]. In [24], the sliding mode control problem for a normalized singular fractional-order system with matched uncertainties was investigated by using linear matrix inequality. Adaptive sliding mode control was presented for a class of fractional-order nonlinear time-delay systems based on fractional-order disturbance observer, and the Gronwall inequality approach is used to ensure that the output tracking error is uniformly bounded for the fractional-order nonlinear system [28].

Notice that several previous works mainly focused on fractional-order neural networks with only single fractional-order derivative of the states. While it is important to introduce an inertial term to obtain the fractional-order inertial model, which is described by fractional differential equations with two different fractional-order derivatives of the state. This models are considered as a powerful tool to produce complicated chaos and bifurcation behavior. Yet, there exists few literatures on fractional-order inertial models reported. In [29], the stability and synchronization for Riemann–Liouville fractional-order time-delayed inertial neural networks are investigated, several feedback controllers were proposed for different cases of fractional-order time-delayed inertial neural networks based on composition properties of Riemann–Liouville fractional-order derivative. Inspired by the analysis above, this paper will focus on finite-time stabilization of fractional-order inertial neural networks. The main contributions of this paper can be summarized as follows:

- (i) Sufficient conditions are obtained to guarantee that the fractional-order inertial neural networks with time delays can be stabilized in finite time.
- (ii) Different from the existing works, the fractional-order inertial neural networks is different from integer-order delayed inertial neural networks models in [12, 13], so those results cannot be directly applied to system given in this paper.
- (iii) The settling time of the finite-time stabilization is estimated, and it is shown theoretically and numerically that the designed feedback controllers are effective.

This article is formulated as follows. In Section 2, some useful definitions, lemmas and model description are presented. The finite-time stabilization result of our model are derived in Section 3. To prove the effectiveness of our results, two examples are given in Section 4. Finally, the conclusion is drawn in section 5.

Notation. $\mathbb{R}$ and $\mathbb{R}^n$ denote the set of real numbers and the n-dimensional Euclidean space for $x = (x_1, \dots, x_n)^{\mathrm{T}} \in \mathbb{R}^n$, respectively, $\operatorname{sign}(\cdot)$ denote the functional sign.

2 Preliminaries and model description

Throughout this paper, the *Caputo* fractional derivative and integral are involved. In this section, we introduce some useful definitions and lemmas. To simplify things, we denote ${}_0^C\mathcal{D}_t^\alpha g(t) = \mathcal{D}_t^\alpha g(t)$.

Consider the following fractional-order differential equation:

$$\begin{aligned} \mathcal{D}_t^\alpha y(t) &= f\big(y(t)\big), \quad y(t) \in \mathbb{R}^n, \\ y(0) &= y_0, \end{aligned}$$

where f is a continuous function such that $f(0) = 0$.

Definition 1. (See [30].) The Caputo fractional derivative with noninteger order $\alpha > 0$ of function $g(t)$ is defined as follows:

$$\mathcal{D}_t^\alpha g(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{1}{(t-\sigma)^{\alpha-n+1}} \left(\frac{\mathrm{d}}{\mathrm{d}\sigma}\right)^n g(\sigma)\,\mathrm{d}\sigma, \quad \alpha > 0,$$

where $n-1 < \alpha < n$, $n \in \mathbb{Z}^+$. Particularly, if $0 < \alpha < 1$,

$$\mathcal{D}_t^\alpha g(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{g'(\sigma)}{(t-\sigma)^\alpha}\,\mathrm{d}\sigma.$$

Proposition 1. (*See* [30].) *For $h_1(t), h_2(t) \in \mathcal{C}^n([0,\infty[, \mathbb{R}^n)$, $\lambda_1, \lambda_2 \in \mathbb{R}$ and $n-1 < \alpha < n$, we have*

$$\mathcal{D}_t^\alpha\bigl(\lambda_1 h_1(t) + \lambda_2 h_2(t)\bigr) = \lambda_1 \mathcal{D}_t^\alpha h_1(t) + \lambda_2 \mathcal{D}_t^\alpha h_2(t).$$

Proposition 2. (*See* [31].) *Let $y(t) \in \mathbb{R}^n$ be a continuous and derivable vector-valued function. Then, for $t \geqslant 0$, we have*

$$\frac{1}{2}\mathcal{D}_t^\alpha y^2(t) \leqslant y(t)\mathcal{D}_t^\alpha y(t), \quad \alpha \in (0,1).$$

Lemma 1. (*See* [27].) *Assume that a continuous, positive-definite function $V(t)$ satisfies the following fractional-order differential inequality*:

$$\mathcal{D}_t^\alpha V(t) \leqslant -cV^\eta(t).$$

Here the constants $c > 0$, $0 < \eta < \alpha$. Then $V(t)$ satisfies the following inequality:

$$V^{\alpha-\eta}(t) \leqslant V^{\alpha-\eta}(t_0) - \frac{c\,\Gamma(1+\alpha-\eta)(t-t_0)^\alpha}{\Gamma(1+\alpha)\Gamma(1-\eta)},$$

and $V(t) = 0$ for all $t \geqslant T$, where T is given as follows:

$$T = t_0 + \left[V^{\alpha-\eta}(t_0)\frac{\Gamma(1+\alpha)\Gamma(1-\eta)}{c\,\Gamma(1+\alpha-\eta)}\right]^{1/\alpha}.$$

Lemma 2. (*See* [5].) *If $z_1, \dots, z_N, q_1, q_2 \in \mathbb{R}$ with $0 < q_1 < q_2$, then the following inequality holds*:

$$\left[\sum_{k=1}^N |z_k|^{q_2}\right]^{1/q_2} \leqslant \left[\sum_{k=1}^N |z_k|^{q_1}\right]^{1/q_1}.$$

2.1 Model description

Consider following Caputo fractional-order inertial neural network with time delays:

$$\mathcal{D}_t^{2\beta} x_i(t) = -a_i \mathcal{D}_t^{\beta} x_i(t) - b_i x_i(t) + \sum_{k=1}^{n} c_{ik} f\bigl(x_k(t)\bigr) + \sum_{k=1}^{n} d_{ik} f_k\bigl(x_k\bigl(t-\tau_k(t)\bigr)\bigr) + I_i(t), \quad t>0. \tag{1}$$

Here $1/2 < \beta < 1$, $n \geqslant 2$ is the amount of units in the neural network, $x_i(\cdot)$ stands for the neuron state, $a_i, b_i > 0$ are constants, $f_k(\cdot)$ denote the activations functions of kth neuron at time t, $\tau_k(\cdot)$ is the time delay with $0 \leqslant \tau_k(\cdot) \leqslant \tau$, c_{ik} and d_{ik} stand for the interconnection weight coefficients of the neurons $1 \leqslant i, k \leqslant n$, $I_i(\cdot)$ is the external input. The initial conditions of system (1) are given by

$$x_i(s) = \check{\varphi}_i(s), \quad \frac{\mathrm{d}x_i(s)}{\mathrm{d}t} = \check{\psi}_i(s), \quad s \in (-\infty, 0],\ i = 1, \dots, n,$$

where $\check{\varphi}_i(\cdot)$ and $\check{\psi}_i(\cdot)$ are real-valued continuous functions on $[-\infty, 0]$. Now, let us introduce the following assumption to derive the main results of this paper:

(A1) For $k = 1, \dots, n$, the activation function f_k satisfies the Lipschitz condition: that is, for $\delta, \tilde{\delta} \in \mathbb{R}$, there exists $M_k > 0$ such that

$$\bigl|f_k(\delta) - f_k(\tilde{\delta})\bigr| \leqslant M_k |\delta - \tilde{\delta}| \quad \text{and} \quad f_k(0) = 0.$$

Let x^* be an equilibrium point of system (1). By a simple transformation $p_i(t) = x_i(t) - x^* \in \mathbb{R}$ we can shift the equilibrium point to the origin. Then system (1) can be rewritten as follows:

$$\mathcal{D}_t^{2\beta} p_i(t) = -a_i \mathcal{D}_t^{\beta} p_i(t) - b_i p_i(t) + \sum_{k=1}^{n} c_{ik} \tilde{f}\bigl(p_k(t)\bigr) + \sum_{k=1}^{n} d_{ik} \tilde{f}_k\bigl(p_k\bigl(t-\tau_k(t)\bigr)\bigr), \tag{2}$$

where $\tilde{f}_k(p_k) = f_k(p_j + x^*) - f_j(x_j^*)$. The initial conditions of system (2) are given by

$$\begin{aligned} p_i(s) &= \check{\varphi}_i(s) - y^* = \varphi(s), \\ \frac{\mathrm{d}z_i(s)}{\mathrm{d}t} &= \check{\psi}_i(s) - y^* = \psi(s), \quad s \in (-\infty, 0],\ i = 1, \dots, n. \end{aligned}$$

Letting $h_i(t) = \mathcal{D}_t^{\beta} p_i(t) + p_i(t)$ for $i = 1, \dots, n$, then system (2) with the control variables can be rewritten as

$$\begin{aligned} \mathcal{D}_t^{\beta} p_i(t) &= -p_i(t) + h_i(t) + U_i(t), \\ \mathcal{D}_t^{\beta} h_i(t) &= -(a_i - 1)h_i(t) - (b_i - a_i + 1)p_i(t) \\ &\quad - \sum_{k=1}^{n} c_{ik} \tilde{f}_k(p_k(t)) + \sum_{k=1}^{n} d_{ik} \tilde{f}_k\bigl(p_j\bigl(t-\tau_k(t)\bigr)\bigr) + \bar{U}_i(t), \end{aligned} \tag{3}$$

where $U(\cdot) = (U_1(\cdot), \dots, U_n(\cdot))^{\mathrm{T}}$ and $\bar{U}(\cdot) = (\bar{U}_1(\cdot), \dots, \bar{U}_n(\cdot))^{\mathrm{T}}$ are the control variables, and the initial conditions become

$$\begin{aligned} &p_i(s) = \varphi(s), \\ &h_i(t) = \varphi(s) + \psi(s), \quad s \in (-\infty, 0],\ i = 1, \dots, n. \end{aligned} \tag{4}$$

Definition 2. System (3) is finite-time stabilizable if under a suitable designed feedback controls $U_i(t)$ and $\bar{U}_i(t)$, there exists T dependent on the initial conditions (4) such that the closed-loop system is finite-time stable.

3 Main results

In this section, the finite-time stabilization of Caputo fractional-order inertial neural networks (CFOINN) with time-delays will be investigated.

Theorem 1. *Assume that* (A1) *holds and the time-varying delay* $\tau_k(\cdot)$ *is known. System* (3) *is finite-time stabilized under the following feedback control law*:

$$\begin{aligned} U_i(t) &= -w_{i1} p_i(t) - w_{i2} \operatorname{sign}\bigl(p_i(t)\bigr)\bigl|p_i(t)\bigr|^{\upsilon}, \\ \bar{U}_i(t) &= -\bar{w}_{i1} h_i(t) - \bar{w}_{i2} \operatorname{sign}\bigl(h_i(t)\bigr)\bigl|h_i(t)\bigr|^{\upsilon} \\ &\quad - \sum_{k=1}^{n} d_{ik} \tilde{f}_k\bigl(p_k\bigl(t - \tau_k(t)\bigr)\bigr), \end{aligned} \tag{5}$$

where

$$2w_{i1} \geqslant |a_i - b_i - 1| - 1 + \sum_{k=1}^{n} |c_{ki}| M_i \tag{6}$$

and

$$2\bar{w}_{i1} \geqslant -2a_i + |a_i - b_i - 1| + \sum_{k=1}^{n} |c_{ik}| M_k + 3, \tag{7}$$

$$w_{i2} > 0, \qquad \bar{w}_{i2} > 0, \qquad 0 < \upsilon < \beta. \tag{8}$$

The settling time stabilization T will be estimated by

$$T = \left[V^{\beta - \frac{1+\upsilon}{2}}(0) \frac{\Gamma(1+\beta)\Gamma(1 - \frac{1+\upsilon}{2})}{\varpi 2^{(\upsilon+1)/2} \Gamma(1 + \beta - \frac{1+\upsilon}{2})} \right]^{1/\beta}, \tag{9}$$

where

$$V(0) = \frac{1}{2} \left[\sum_{i=1}^{n} h_i^2(0) + \sum_{i=1}^{n} p_i^2(0) \right], \qquad \varpi = \min\Bigl\{ \min_{1 \leqslant i \leqslant n} \{w_{i2}\}, \min_{1 \leqslant i \leqslant n} \{\bar{w}_{i2}\} \Bigr\}.$$

Proof. Let us choose the following Lyapunov function candidate:

$$V(t) = \frac{1}{2} \left[\sum_{i=1}^{n} h_i^2(t) + \sum_{i=1}^{n} p_i^2(t) \right]. \tag{10}$$

By Propositions 1 and 2 the time derivative of (10) along the trajectories of system (3) can be calculated as follows:

$$\begin{aligned}
\mathcal{D}_t^{\beta} V(t) &= \mathcal{D}_t^{\beta} \frac{1}{2}\left[\sum_{i=1}^{n} h_i^2(t) + \sum_{i=1}^{n} p_i^2(t)\right] = \frac{1}{2}\sum_{i=1}^{n} \mathcal{D}_t^{\beta} h_i^2(t) + \frac{1}{2}\sum_{i=1}^{n} \mathcal{D}_t^{\beta} p_i^2(t) \\
&\leqslant \sum_{i=1}^{n} \left[h_i(t)\mathcal{D}_t^{\beta} h_i(t) + p_i(t)\mathcal{D}_t^{\beta} p_i(t)\right] \\
&\leqslant \sum_{i=1}^{n} \Biggl[-(a_i-1)h_i^2(t) - (b_i - a_i + 1)p_i(t)h_i(t) \\
&\quad + \sum_{k=1}^{n} c_{ik}\tilde{f}_k\bigl(p_k(t)\bigr)h_i(t) - \bar{w}_{i1}h_i^2(t) - \bar{w}_{i2}\bigl|h_i(t)\bigr|^{\upsilon+1} \\
&\quad - p_i^2(t) + h_i(t)p_i(t) - w_{i1}p_i^2(t) - w_{i2}\bigl|p_i(t)\bigr|^{\upsilon+1}\Biggr] \\
&\leqslant \sum_{i=1}^{n} \Biggl[-(a_i-1)h_i^2(t) + |a_i - b_i - 1|\bigl|p_i(t)\bigr|\bigl|h_i(t)\bigr| \\
&\quad + \sum_{k=1}^{n} |c_{ik}|\bigl|\tilde{f}_k\bigl(p_k(t)\bigr)\bigr|\bigl|h_i(t)\bigr| - \bar{w}_{i1}h_i^2(t) - \bar{w}_{i2}\bigl|h_i(t)\bigr|^{\upsilon+1} \\
&\quad - p_i^2(t) + \bigl|h_i(t)\bigr|\bigl|p_i(t)\bigr| - w_{i1}p_i^2(t) - w_{i2}\bigl|p_i(t)\bigr|^{\upsilon+1}\Biggr].
\end{aligned}$$

Since we have $2xy \leqslant x^2 + y^2$ for all $x, y \in \mathbb{R}^+$, it follows that

$$\bigl|p_i(t)\bigr|\bigl|h_i(t)\bigr| \leqslant \frac{1}{2}p_i^2(t) + \frac{1}{2}h_i^2(t).$$

It follows that

$$\begin{aligned}
\mathcal{D}_t^{\beta} V(t) \leqslant \sum_{i=1}^{n} \Biggl\{ &\left(1 - a_i + \frac{|a_i - b_i - 1|}{2} + \frac{1}{2}\sum_{k=1}^{n} |c_{ik}|M_k + \frac{1}{2} - \bar{w}_{i1}\right) h_i^2(t) \\
&+ \left(\frac{|a_i - b_i - 1|}{2} - \frac{1}{2} + \frac{1}{2}\sum_{k=1}^{n} |c_{ki}|M_i - w_{i1}\right) p_i^2(t) \\
&- \bar{w}_{i2}\bigl|h_i(t)\bigr|^{\upsilon+1} - w_{i2}\bigl|p_i(t)\bigr|^{\upsilon+1}\Biggr\}.
\end{aligned}$$

From (6) and (7) we obtain

$$\mathcal{D}_t^{\beta} V(t) \leqslant \sum_{i=1}^{n} \bigl\{-\bar{w}_{i2}|h_i(t)|^{\upsilon+1} - w_{i2}\bigl|p_i(t)\bigr|^{\upsilon+1}\bigr\}.$$

Since $0 < \upsilon + 1 < 2$, from Lemma 2 we have

$$-\left[\sum_{j=1}^{n} \left|p_j(t)\right|^{\upsilon+1}\right]^{1/(\upsilon+1)} \leqslant -\left[\sum_{j=1}^{n} \left|p_j(t)\right|^{2}\right]^{1/2}$$

and

$$-\left[\sum_{j=1}^{n} \left|h_j(t)\right|^{\upsilon+1}\right]^{1/(\upsilon+1)} \leqslant -\left[\sum_{j=1}^{n} \left|h_j(t)\right|^{2}\right]^{1/2}.$$

So,

$$-\left[\sum_{j=1}^{n} \left|p_j(t)\right|^{\upsilon+1}\right] \leqslant -\left[\sum_{j=1}^{n} \left|p_j(t)\right|^{2}\right]^{(\upsilon+1)/2}$$

and

$$-\left[\sum_{j=1}^{n} \left|h_j(t)\right|^{\upsilon+1}\right] \leqslant -\left[\sum_{j=1}^{n} \left|h_j(t)\right|^{2}\right]^{(\upsilon+1)/2}.$$

Then we obtain

$$\mathcal{D}_t^{\beta} V(t) \leqslant -\min\Big\{\min_{1\leqslant i\leqslant n}\{w_{i2}\}, \min_{1\leqslant i\leqslant n}\{\bar{w}_{i2}\}\Big\} 2^{(\upsilon+1)/2} V^{(\upsilon+1)/2}(t).$$

That is,

$$\mathcal{D}_t^{\beta} V(t) \leqslant -\varpi 2^{(\upsilon+1)/2} V^{(\upsilon+1)/2}(t).$$

Since we have $0 < \upsilon < \beta$ and $1/2 < \beta < 1$, then $0 < \beta - (\upsilon + 1)/2$. So, $(\upsilon + 1)/2 < \beta$. Therefore, from Lemma 1, system (3) is finite-time stabilizable, and the settling time T is given by (9). □

In Theorem 1, by designing a special fixed-time controller we achieved the fixed-time stabilization of system (3). However, $\bar{U}_i$ in (5) is a delay dependent feedback control, which is not suitable for real-world applications. Thus, we seek to obtain a fixed-time controller that is more suitable in practice and able to stabilize in fixed-time the FOINNs (3). To this end, we need to impose the boundedness of our activation functions:

(A2) There exists $L_k \in \mathbb{R}_+^*$ such that

$$\left|\tilde{f}_k(\cdot)\right| \leqslant L_k \quad \text{for } k = 1, \dots, n.$$

We have the following result:

Theorem 2. *If assumptions* (A1) *and* (A2) *hold and conditions* (6), (7) *and* (8) *are satisfied, let*

$$\bar{w}_{i3} \geqslant \sum_{k=1}^{n} |d_{ik}| L_k. \tag{11}$$

Then system (3) *is finite-time stabilizable under the feedback control law*

$$\begin{aligned} U_i(t) &= -w_{i1}p_i(t) - w_{i2}\,\mathrm{sign}\big(p_i(t)\big)\big|p_i(t)\big|^{\upsilon}, \\ \bar{U}_i(t) &= -\bar{w}_{i1}h_i(t) - \bar{w}_{i2}\,\mathrm{sign}\big(h_i(t)\big)\big|h_i(t)\big|^{\upsilon} - \bar{w}_{i3}\,\mathrm{sign}\big(h_i(t)\big), \end{aligned} \tag{12}$$

and the settling time is expressed as (9)*.*

Proof. Let us choose the following Lyapunov function candidate:

$$V(t) = \frac{1}{2}\left[\sum_{i=1}^{n} h_i^2(t) + \sum_{i=1}^{n} p_i^2(t)\right]. \tag{13}$$

By Propositions 1 and 2 the time derivative of (13) along the trajectories of system (3) can be calculated as follows:

$$\begin{aligned} \mathcal{D}_t^{\beta} V(t) &= \mathcal{D}_t^{\beta} \frac{1}{2}\left[\sum_{i=1}^{n} h_i^2(t) + \sum_{i=1}^{n} p_i^2(t)\right] = \frac{1}{2}\sum_{i=1}^{n} \mathcal{D}_t^{\beta} h_i^2(t) + \frac{1}{2}\sum_{i=1}^{n} \mathcal{D}_t^{\beta} p_i^2(t) \\ &\leqslant \sum_{i=1}^{n} \big[h_i(t)\mathcal{D}_t^{\beta} h_i(t) + p_i(t)\mathcal{D}_t^{\beta} p_i(t)\big] \\ &\leqslant \sum_{i=1}^{n} \Bigg[-(a_i-1)h_i^2(t) + |a_i - b_i - 1|p_i(t)h_i(t) \\ &\quad + \sum_{k=1}^{n} |c_{ik}|\big|\tilde{f}_k\big(p_k(t)\big)\big|h_i(t) + \sum_{k=1}^{n} |d_{ik}|\big|\tilde{f}_k\big(p_k\big(t-\tau_k(t)\big)\big)\big|\big|h_i(t)\big| \\ &\quad - \bar{w}_{i1}h_i^2(t) - \bar{w}_{i2}\big|h_i(t)\big|^{\upsilon+1} - \bar{w}_{i3}\big|h_i(t)\big| \\ &\quad - p_i^2(t) + h_i(t)p_i(t) - w_{i1}p_i^2(t) - w_{i2}\big|p_i(t)\big|^{\upsilon+1}\Bigg]. \end{aligned}$$

From (A1) and (A2) we obtain

$$\begin{aligned} \mathcal{D}_t^{\beta} V(t) &\leqslant \sum_{i=1}^{n} \Bigg[-(a_i-1)h_i^2(t) + |a_i - b_i - 1|p_i(t)h_i(t) \\ &\quad + \sum_{k=1}^{n} |c_{ik}|M_k\big|\big(p_k(t)\big)\big|h_i(t) + \sum_{k=1}^{n} |d_{ik}|L_k\big|h_i(t)\big| \\ &\quad - \bar{w}_{i1}h_i^2(t) - \bar{w}_{i2}\big|h_i(t)\big|^{\upsilon+1} - \bar{w}_{i3}\big|h_i(t)\big| \\ &\quad - p_i^2(t) + h_i(t)p_i(t) - w_{i1}p_i^2(t) - w_{i2}\big|p_i(t)\big|^{\upsilon+1}\Bigg] \\ &\leqslant \sum_{i=1}^{n} \Bigg\{\Bigg(1 - a_i + \frac{|a_i - b_i - 1|}{2} + \frac{1}{2}\sum_{k=1}^{n} |c_{ik}|M_k + \frac{1}{2} - \bar{w}_{i1}\Bigg)h_i^2(t) \end{aligned}$$

$$
\begin{aligned}
&+\left(\sum_{k=1}^{n}|d_{ik}|L_k-\bar{w}_{i3}\right)\left|h_i(t)\right| \\
&+\left(\frac{|a_i-b_i-1|}{2}-\frac{1}{2}+\frac{1}{2}\sum_{k=1}^{n}|c_{ki}|M_i-w_{i1}\right)p_i^2(t) \\
&-\bar{w}_{i2}\left|h_i(t)\right|^{\upsilon+1}-w_{i2}\left|p_i(t)\right|^{\upsilon+1}\Biggr\}.
\end{aligned}
$$

From (6), (7) and (11) we get

$$
\mathcal{D}_t^{\beta}V(t)\leqslant\sum_{i=1}^{n}\left\{-\bar{w}_{i2}\left|h_i(t)\right|^{\upsilon+1}-w_{i2}\left|p_i(t)\right|^{\upsilon+1}\right\}.
$$

At this stage, we notice that the delays have been removed from the calculus. So, the proof can be continued as for a delay-free system. We have

$$
\mathcal{D}_t^{\beta}V(t)\leqslant-\min\Bigl\{\min_{1\leqslant i\leqslant n}\{w_{i2}\},\min_{1\leqslant i\leqslant n}\{\bar{w}_{i2}\}\Bigr\}2^{(\upsilon+1)/2}V^{(\upsilon+1)/2}(t).
$$

That is,

$$
\mathcal{D}_t^{\beta}V(t)\leqslant-\varpi 2^{(\upsilon+1)/2}V^{(\upsilon+1)/2}(t).
$$

Therefore, from Lemma 1, system (3) is finite-time stable, and the settling time T is given by (9). □

Remark 1. It should be noted that the setting time T can be theoretically determined according to the equality 9 in Theorems 1 and 2. In 9, we see that the setting time T not only depends on the initial states $h(0)$ and $p(0)$, but also depends on the fractional order β, parameters υ and ϖ.

Remark 2. Theoretically, there are no restrictions on activations functions $\tilde{f}_i$ ($i=1,\dots,n$) and time delays in Theorems 1 and 2. But from the view of engineering the functions $\tilde{f}_i$ ($i=1,\dots,n$) and $\tau(\cdot)$ are needed to be known in advance, which is the main limitation of our theoretical results and needs to be relaxed in the future work.

4 Numerical examples

To illustrate the effectiveness of our results, two examples are presented in this section.

Example 1. Consider the following CFOINN for $i=1,2,3$:

$$
\begin{aligned}
\mathcal{D}_t^{2\beta}p_i(t)&=-a_i\mathcal{D}_t^{\beta}p_i(t)-b_ip_i(t) \\
&\quad+\sum_{k=1}^{3}c_{ik}f_k\bigl(p_k(t)\bigr)+\sum_{k=1}^{3}d_{ik}f_k\bigl(p_k(t-\tau_k)\bigr)+I_i.
\end{aligned}\tag{14}
$$

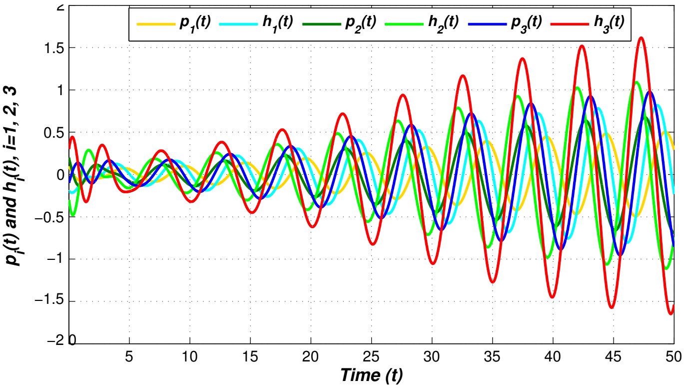

Figure 1. Trajectories state of system (15) without control.

The system parameters are set as follows:

$$a_1 = a_2 = a_3 = 1.01, \qquad b_1 = b_2 = b_3 = 4.01,$$

$$\upsilon = 0.6, \qquad \tilde{f}_k(\cdot) = \sin(\cdot),$$

$$I_1 = I_2 = I_3 = 0, \qquad \beta = 0.98,$$

$$C = \begin{pmatrix} 1.6 & -1.5 & 2.8 \\ -2 & -2.1 & 1.5 \\ 1.9 & 2 & 1.7 \end{pmatrix}, \qquad D = \begin{pmatrix} 1 & -1.5 & -2 \\ -1.5 & 2 & -0.5 \\ -2 & 2 & -1.5 \end{pmatrix}.$$

Letting $h_i(t) = \mathcal{D}_t^\beta p_i(t) + p_i(t)$, $i = 1, 2, 3$, system (14) with control can be rewritten as

$$\begin{aligned} \mathcal{D}_t^\beta p_i(t) &= -p_i(t) + h_i(t) + U_i(t), \\ \mathcal{D}_t^\beta h_i(t) &= -(a_i - 1)h_i(t) - (b_i - a_i + 1)p_i(t) \\ &\quad + \sum_{k=1}^{3} c_{ik}\tilde{f}_k\bigl(p_k(t)\bigr) + \sum_{k=1}^{3} d_{ik}\tilde{f}_k\bigl(p_k\bigl(t - \tau_k(t)\bigr)\bigr) + \bar{U}_i. \end{aligned} \tag{15}$$

The state trajectories of system (15) without control is depicted in Fig. 1. Afterwards, according to the conditions presented in Theorem 1, we choose

$$w_{11} = 5 \geqslant \frac{1}{2}|a_1 - b_1 - 1| - \frac{1}{2} + \frac{1}{2}\sum_{k=1}^{3}|c_{k1}|,$$

$$w_{21} = 5 \geqslant \frac{1}{2}|a_2 - b_2 - 1| - \frac{1}{2} + \frac{1}{2}\sum_{k=1}^{3}|c_{k2}|,$$

$$w_{31} = 5 \geqslant \frac{1}{2}|a_3 - b_3 - 1| - \frac{1}{2} + \frac{1}{2}\sum_{k=1}^{3}|c_{k3}|$$

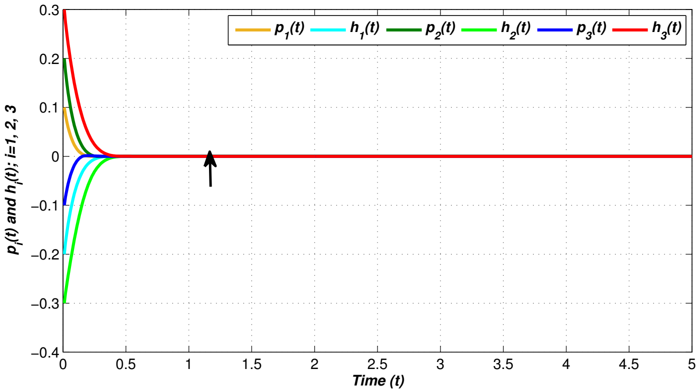

Figure 2. Trajectories state of system (15) under control (5).

and

$$\bar{w}_{11} = 6 \geqslant -a_1 + \frac{1}{2}|a_1 - b_1 - 1| + \frac{1}{2}\sum_{k=1}^{3}|c_{1k}| + \frac{3}{2},$$
$$\bar{w}_{21} = 5.5 \geqslant -a_2 + \frac{1}{2}|a_2 - b_2 - 1| + \frac{1}{2}\sum_{k=1}^{3}|c_{2k}| + \frac{3}{2},$$
$$\bar{w}_{31} = 5.5 \geqslant -a_3 + \frac{1}{2}|a_3 - b_3 - 1| + \frac{1}{2}\sum_{k=1}^{3}|c_{3k}| + \frac{3}{2}.$$

The Lipschitz condition is $M_k = 1$, and if we choose the parameters $w_{12} = 2 > 0$, $w_{22} = 1.7 > 0$, $w_{32} = 2 > 0$, $\bar{w}_{12} = 1.7 > 0$, $\bar{w}_{22} = 1.8 > 0$, $\bar{w}_{32} = 1.8 > 0$, all condition of Theorem 1 are justified. Then system (15) is finite-time stabilizable by controller (5). Taking the initial values as

$$p_1(0) = 0.1, \qquad p_2(0) = 0.2, \qquad p_3(0) = -0.2,$$
$$h_1(0) = -0.1, \qquad h_2(0) = -0.3, \qquad h_3(0) = 0.3.$$

The settling time is estimated by

$$T = \left[V^{0.98-(1+0.6)/2}(0)\frac{\Gamma(1+0.98)\Gamma(1-\frac{1+0.6}{2})}{\varpi 2^{(0.6+1)/2}\Gamma(1+0.98-\frac{1+0.6}{2})}\right]^{1/0.98} = 1.1725$$

with

$$V(0) = \frac{1}{2}\left[\sum_{i=1}^{n} h_i^2(0) + \sum_{i=1}^{n} p_i^2(0)\right] = 0.14,$$
$$\varpi = \min\left\{\min_{1\leqslant i\leqslant 3}\{w_{i2}\}, \min_{1\leqslant i\leqslant 3}\{\bar{w}_{i2}\}\right\} = 1.7.$$

Trajectories states of system (15) under control (12) are shown in Fig. 2.

Remark 3. Up to now, stability and synchronization for integer-order inertial neural networks with delay have been intensively studied [13]. In this paper, finite-time stabilization for fractional-order inertial neural networks with time-varying delays are investigated. Note that when $\beta = 1$, fractional-order inertial neural network will be reduced to integer-order inertial neural network. Therefore, integer-order inertial neural network can be seen as a special case of fractional-order inertial neural networks. The model proposed in this paper is less conservative and more general.

Remark 4. Noting that there are few results dealing with the finite-time stability or synchronization for FONNs [25, 26]. In [26], authors investigated the problem of finite-time Mittag-Leffler synchronization of FONNs by mean of Laplace transform and the generalized Gronwall inequality. In [25], Xiao et al. studied the finite-time Mittag-Leffler synchronization of a class of fractional memristive BAM neural networks with time delays. It should be noticed that the authors do not give the value of setting time T. In this paper, based on Lemma 1 and Lyapunov theory, we investigate the finite-time stabilization of fractional-order inertial neural network and give the value of the setting time T. Compared with previous works [25,26], our result obtained in this paper has better application. On the other hand, our study offers an improvement compared with [13], where only asymptotic and exponential stability of neural networks are considered.

Example 2. Now, consider the following CFOINN for $i = 1, \dots, 3$:

$$\begin{aligned} \mathcal{D}_t^{2\beta} p_i(t) &= -a_i \mathcal{D}_t^{\beta} p_i(t) - b_i p_i(t) \\ &\quad + \sum_{k=1}^{3} c_{ik} f_k\bigl(p_k(t)\bigr) + \sum_{k=1}^{3} d_{ik} f_k\bigl(p_k(t - \tau_k)\bigr) + I_i, \end{aligned}$$

where

$$a_1 = a_2 = a_3 = 1.02, \qquad b_1 = b_2 = b_3 = 4.02,$$
$$\tau = 0.9, \qquad \upsilon = 0.5, \qquad \tilde{f}_k(\cdot) = \tanh(\cdot),$$
$$I_1 = I_2 = I_3 = 2, \qquad \beta = 0.99, \qquad M_k = 1, \qquad L_k = 1,$$
$$C = \begin{pmatrix} 1.5 & 1.4 & 2.7 \\ -1.9 & -2 & 1.4 \\ 1.8 & 1.9 & 1.6 \end{pmatrix}, \qquad D = \begin{pmatrix} 0.9 & -1.4 & -1.9 \\ -1.4 & 1.9 & -0.4 \\ -1.9 & 1.9 & -1.4 \end{pmatrix}.$$

Let $h_i(t) = \mathcal{D}_t^{\beta} p_i(t) + p_i(t)$ for $i = 1, \dots, 3$, then system (14) with control can be rewritten as

$$\begin{aligned} \mathcal{D}_t^{\beta} p_i(t) &= -p_i(t) + h_i(t) + U_i(t), \\ \mathcal{D}_t^{\beta} h_i(t) &= -(a_i - 1) h_i(t) - (b_i - a_i + 1) p_i(t) \\ &\quad + \sum_{k=1}^{3} c_{ik} \tilde{f}_k\bigl(p_k(t)\bigr) + \sum_{k=1}^{3} d_{ik} \tilde{f}_k\bigl(p_k\bigl(t - \tau_k(t)\bigr)\bigr) + \bar{U}_i. \end{aligned} \tag{16}$$

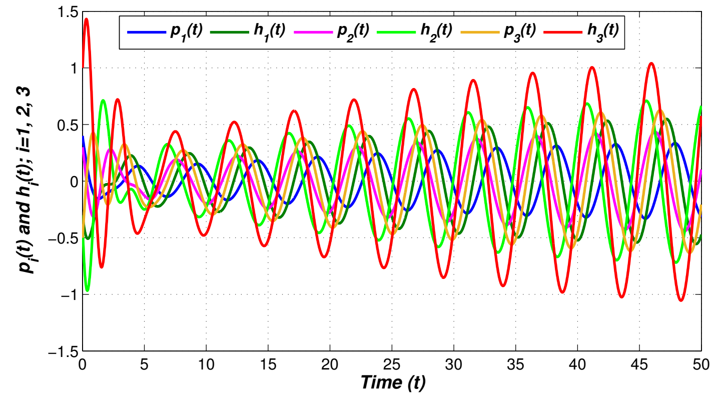

Figure 3. Trajectories state of system (16) without control.

The sate trajectories of system (16) without control input is shown in Fig. 3. According to the conditions presented in Theorem 2, we choose

$$w_{11} = 4.5 \geqslant \frac{1}{2}|a_1 - b_1 - 1| - \frac{1}{2} + \frac{1}{2}\sum_{k=1}^{3}|c_{k1}| = 4.1,$$

$$w_{21} = 4.5 \geqslant \frac{1}{2}|a_2 - b_2 - 1| - \frac{1}{2} + \frac{1}{2}\sum_{k=1}^{3}|c_{k2}| = 4.2,$$

$$w_{31} = 4.5 \geqslant \frac{1}{2}|a_3 - b_3 - 1| - \frac{1}{2} + \frac{1}{2}\sum_{k=1}^{3}|c_{k3}| = 4.35,$$

$$\bar{w}_{11} = 5.5 \geqslant -a_1 + \frac{1}{2}|a_1 - b_1 - 1| + \frac{1}{2}\sum_{k=1}^{3}|c_{1k}| + \frac{3}{2} = 5.28,$$

$$\bar{w}_{21} = 5.5 \geqslant -a_2 + \frac{1}{2}|a_2 - b_2 - 1| + \frac{1}{2}\sum_{k=1}^{3}|c_{2k}| + \frac{3}{2} = 5.13,$$

$$\bar{w}_{31} = 5.5 \geqslant -a_3 + \frac{1}{2}|a_3 - b_3 - 1| + \frac{1}{2}\sum_{k=1}^{3}|c_{3k}| + \frac{3}{2} = 5.13$$

and

$$\bar{w}_{13} = 4.5 \geqslant \sum_{k=1}^{3}|d_{1k}|L_k = 4.2, \qquad \bar{w}_{23} = 4 \geqslant \sum_{k=1}^{3}|d_{2k}|L_k = 3.7,$$

$$\bar{w}_{33} = 5.5 \geqslant \sum_{k=1}^{3}|d_{3k}|L_k = 5.2.$$

The Lipschitz condition is $M_k = 1$, $L_k = 1$, and if we choose the parameters $w_{12} = 1.8 > 0$, $w_{22} = 1.8 > 0$, $w_{32} = 1.5 > 0$, $\bar{w}_{12} = 1.7 > 0$, $\bar{w}_{22} = 1.6 > 0$, $\bar{w}_{32} = 1.7 > 0$,

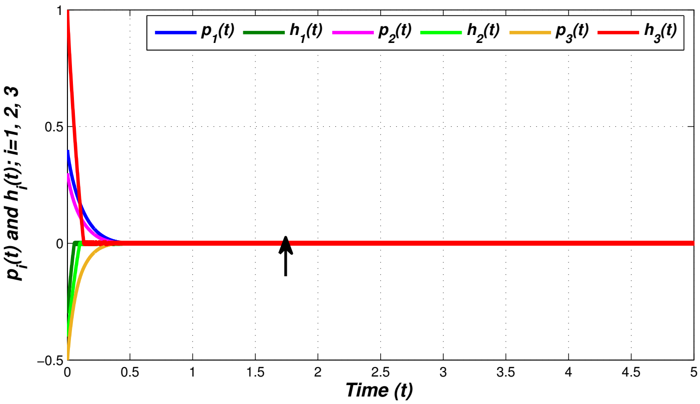

Figure 4. Trajectories state of system (16) under control (12).

then system (16) is finite-time stabilizable by controller (12). Taking the initial values as

$$\begin{aligned} &p_1(0) = 0.4, && p_2(0) = 0.3, && p_3(0) = -0.5, \\ &h_1(0) = -0.3, && h_2(0) = -0.4, && h_3(0) = 1, \end{aligned}$$

the settling time is estimated by

$$T = \left[V^{0.99-(1+0.5)/2}(0) \frac{\Gamma(1+0.99)\Gamma(1-\frac{1+0.5}{2})}{\varpi 2^{(0.5+1)/2}\Gamma(1+0.99-\frac{1+0.5}{2})} \right]^{1/0.99} = 1.7352$$

with

$$\begin{aligned} V(0) &= \frac{1}{2}\left[\sum_{i=1}^{n} h_i^2(0) + \sum_{i=1}^{n} p_i^2(0) \right] = 1.25, \\ \varpi &= \min\left\{ \min_{1\leqslant i\leqslant 3}\{w_{i2}\}, \min_{1\leqslant i\leqslant 3}\{\bar{w}_{i2}\} \right\} = 1.5. \end{aligned}$$

The states trajectories are depicted in Fig. 4.

Remark 5. Our results on finite-time stabilization are derived based on the Lyapunov direct method and feedback controller, which are more concise and easy to verify than those obtained in the existing papers by using linear matrix inequality [24, 32], the matrix measure method and Halany inequality techniques [12].

5 Conclusion

In this paper, the finite-time stabilization of fractional-order inertial neural networks with time delays has been investigated. Based on a novel nonlinear feedback controller, finite-time stability theory and some inequality, we established a new criterion of finite-time stabilization of. The validity of the proposed controllers and the effectiveness of the obtained results have been illustrated by some numerical examples. To the best of our knowledge, this is the first paper to study the finite-time stabilization for FOINNs with time-varying delays and give the value of the setting time. The future work mainly includes the following aspects:

1. How to analyze the finite-time stabilization of more complex fractional-order neural network model with stochastic perturbation and various time delays such as time-varying delays, infinite distributed delays and neutral-type delays.
2. How to derive the fixed-time stabilization conditions of fractional-order neural network model.
3. How to deal with the problem of finite-time stabilization via different types of controllers.

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