



# Finite-time matrix projective synchronization of fractional-order memristor-based delayed neural networks with parameter uncertainty\*

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**Received:** June 28, 2024 / **Revised:** November 2, 2025 / **Published online:** February 07, 2026

**Abstract.** This research examines the phenomenon of finite-time matrix projective synchronization (FTMPS) in the context of two distinct fractional-order memristor-based delayed neural networks (FOMDNNs). For the FOMDNNs with indeterminate parameters, some suitable controllers are structured, and sufficient conditions for implementing the FTMPS are demonstrated through some inequality techniques and relevant lemmas pertaining to fractional calculus. The synchronization issues under two different norm cases are fully considered. Subsequently, two numerical examples of the FTMPS are revealed and the discrepancies in their synchronization gradually tend to zero, which shows the validity and accuracy of the obtained synchronization results.

**Keywords:** Caputo derivative, finite-time synchronization, parameter uncertainty, fractional-order memristive-based delayed neural networks.

## 1 Introduction

In recent decades, the growth and research of various dynamical behaviors of neural networks (NNs) have changed with each passing days and have garnered increasing attention

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\*This work was supported by the National Natural Science Foundation of China (No. 61833005) and the Natural Science Foundation of Anhui Province of China (No. 1908085MA01).

from researchers. The neural network within the human brain is a sophisticated and intricate system. Furthermore, NNs are extensively employed in various domains, including automatic control, optimization, signal processing, and image processing [28]. Consequently, it is both essential and pragmatic to investigate the dynamic behavior of NNs. A considerable quantity of NNs has been developed, including BAMNNs [3], Cohen–Grossberg NNs [31], fuzzy NNs [4], and so on.

Due to the imperfect conditions of the external environment, various external factors can prompt changes or even chaos in the nervous system. By introducing time delays and uncertain parameters, NNs can better simulate and adapt to the dynamic changes and uncertainties in the real world, thereby improving the system's robustness, fault tolerance, and enhancing the model's predictive accuracy in practical applications to address more complex application issues. Uncertainty has the potential to influence the synchronization, stability, and various other dynamic behaviors of systems [14, 29]. Hu et al. [14] investigated the state estimation issue of FOCVNNs with uncertain parameters and time delays, employing a nonresolution approach and Lyapunov function. In [29], utilizing the Laplace transform in conjunction with nonlinear control methodologies, Zhang et al. achieved quasiprojection synchronization (QPS) in Caputo inertial time-delay NNs. Because of that, it is indispensable and momentous to explore the FTMPS of FODMNNs with uncertain parameters.

Memristor was first put forth by Chua [5] and realized by HP Labs [21]. It was considered as a component of the relationship between circuits. Researchers have identified that memristors possess significant advantages, including high integration capabilities [10], robustness, and fault tolerance [1], which can be attributed to their nanoscale dimensions and passive device characteristics. On account of the application foreground of memristors, an increasing number of researchers are actively seeking this type of device [15].

In comparison to integer calculus, fractional calculus has a greater number of independent variables and infinite memory. Fractional calculus is introduced into many common integer NNs to turn into new fractional-order NNs (FONNs). In addition, some dynamical behaviors are also worthy of exploration by scholars, including bifurcation [27], state estimation [14], and synchronization [29, 30].

Synchronization of NNs is an important and vital dynamical property. There exist a variety of synchronization, comprising finite/fixed-time synchronization, antisynchronization, lag synchronization, global synchronization, predefined-time synchronization, projective synchronization (PS), and so on [18, 23]. PS, that is, the motion path of the master-slave systems will gradually reach consensus under the prerequisite of a scale factor, is a vital conception in practical situations. Currently, numerous scholars have incorporated PS into FONNs. In [7], researchers studied the lag projective synchronization of nonidentical FODMNNs by using the principle of differential inclusion and constructing a sliding mode surface. Besides, the change of projective scale factor can form a variety of synchronizations, for example, MPS [12], modified PS [20], antisynchronization, and CS [33]. In the aforementioned study of the dynamic behavior of NNs, many authors have only focused on research conducted over infinite time. However, in practical applications, the synchronous behavior of NNs over FT is often of greater concern. So far, there

exist two distinct categories of FTS. The first FTS genre in [8] refers to keeping the synchronization error within a small range over a certain time interval, which is more realistic and thus termed practical FTS. The second genre of FTS was defined in reference [26], it is pointed out that the error approaches zero within a limited duration. This paper studies the latter type of FTS problems.

Considering the aforementioned context, research on FOMNNs predominantly focuses on infinite-time intervals, which may not adequately reflect the practical applications and requirements of the real world. Consequently, investigating the potential for achieving synchronization within a finite-time frame is of considerable importance. Furthermore, the distinctions among CS, lag synchronization, and modified PS within neural systems can be attributed to the continuous variation of the projection scale factor. Thus, identifying a more generalized projection scale factor that encompasses these various synchronization concepts represents a significant area of inquiry. The matrix projection scale factor introduced in this study is informed by these considerations; however, it also presents certain challenges, such as effectively addressing the transformation of the matrix scaling factor within the error system. The following sections outline several innovations presented in this paper:

- Drawing upon the work of [7], a more pragmatic model of FOMNNs is introduced, which takes into account both delays and the uncertainties associated with system parameters.
- Unlike [2], this article inquires into the problems of FTMPs. The projective scale factor is extended from constant to constant matrix, which is more universal than the existing conclusions.
- Under the conditions of different norms, it is proved that the selected controllers can attain the FTMPs of the master-slave systems.
- Under the conditions that the scale factor is a constant matrix, two FTS criteria for FOMNNs with parameter uncertainties have been established. Meanwhile, the settling times are validly estimated.

The following is an overview of the writing structure for the subsequent content of this article. Section 2 encompasses the definitions, lemmas, and pertinent assumptions necessary for the discourse presented in this article. The neural network model and error system studied in this article are detailed in Section 3. Section 4 discusses two theorems regarding the system's implementation of FTS. Section 5 provides two numerical examples to corroborate the findings presented in this article. Finally, a summary is provided in Section 6.

## 2 Preliminaries

**Notations.** Through this paper,  $\mathbb{R}$  is the space of real numbers,  $\mathbb{R}^n$  denotes the  $n$ -dimensional Euclidean space.  $\text{sign}(\cdot)$  is the sign function.  $\text{diag}\{\dots\}$  denotes a diagonal matrix. In addition,  $\mathcal{C}^1([t_0, +\infty), \mathbb{R})$  denotes the space consisting of continuous differentiable functions from  $[t_0, +\infty)$  into  $\mathbb{R}$ .

**Definition 1.** (See [19].) Let  $m(t)$  be a function on  $[t_0, +\infty)$  that is differentiable in the first order. Then Caputo fractional derivative of  $m(t)$  is formally defined as follows:

$${}_{t_0}^C D_t^\alpha m(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^t \frac{m'(\eta)}{(t-\eta)^\alpha} d\eta, \quad t \geq t_0,$$

where the order  $0 < \alpha < 1$ .

**Definition 2.** The FOMNN master system  $s_c(t)$  and slave system  $o_c(t)$  are FTMTTP for any initial value if there exists a constant matrix  $\nabla = (\nabla_{ck})_{\mathbb{N} \times \mathbb{N}}$  such that

$$\lim_{t \rightarrow t^*} \|g(t)\| = \|s_c - \nabla \cdot o_c(t)\| = 0,$$

and  $g(t) \equiv 0$  for any  $t \geq t^*$ .

**Lemma 1.** (See [32].) If  $\pi(t) \in \mathbb{C}^1([t_0, +\infty), \mathbb{R})$ , one has

$${}_{t_0}^C D_t^\alpha |\pi(t^+)| \leq \text{sign}(\pi(t)) {}_{t_0}^C D_t^\alpha \pi(t) \quad 0 < \alpha < 1.$$

**Lemma 2.** (See [6].) If  $0 < \alpha < 1$  and  $\Omega(t) : [t_0, +\infty) \rightarrow \mathbb{R}^+$  is continuous and differentiable, then

$${}_{t_0}^C D_t^\alpha \Omega^\zeta(t) \leq \zeta \Omega^{\alpha-1}(t) {}_{t_0}^C D_t^\alpha \Omega(t),$$

where  $t \geq t_0$  and  $\zeta \geq 1$ .

**Lemma 3.** (See [11].) Let  $s_1, s_2, \dots, s_{\mathbb{N}} > 0, \iota > 1$ , and  $0 < h \leq 1$ . Then the subsequent pair of inequalities are valid:

$$\sum_{c=1}^{\mathbb{N}} s_c^h \geq \left( \sum_{c=1}^{\mathbb{N}} s_c \right)^h, \quad \sum_{c=1}^{\mathbb{N}} s_c^\iota \geq \mathbb{N}^{1-\iota} \left( \sum_{c=1}^{\mathbb{N}} s_c \right)^\iota.$$

**Lemma 4.** (See [9].) If function  $V(t) \in \mathbb{C}^1([t_0, +\infty), \mathbb{R}^+)$  is positive definite and

$${}_{t_0}^C D_t^\alpha V(t) \leq -bV^{-\xi}(t) - aV^{-\eta}(t), \quad V(t) \in \mathbb{R}^+ \setminus \{0\},$$

where  $0 < \alpha < 1, a, b > 0, \xi \geq 0$ , and  $\xi < \eta < 1 + 2\xi$ , then one has  $\lim_{t \rightarrow t^*} V(t) = 0$  and  $V(t) = 0, t \geq t^*$ , where

$$\begin{aligned} t^* &\leq T \\ &= \left[ \frac{\Gamma(1+\alpha)}{a2^{(\eta-2\xi-1)/(1+\xi)}(1+\eta)} \left( \left( V^{1+\xi}(t_0) + \left(\frac{a}{b}\right)^{(1+\xi)/(\eta-\xi)} \right)^{(1+\eta)/(1+\xi)} - \left(\frac{a}{b}\right)^{(1+\eta)/(\eta-\xi)} \right) \right]^{1/\alpha} + t_0. \end{aligned}$$

**Assumption 1.** (See [29].) For all  $v, \bar{v} \in \mathbb{R}$ , there exists  $H_c > 0$  such that

$$|\bar{h}_c(v) - \bar{h}_c(\bar{v})| \leq H_c |v - \bar{v}|.$$

### 3 Model description

Next, consider the drive system of FOMDNNs as follows:

$$\begin{aligned}
 {}^C D_t^\alpha o_c(t) = & -d_c o_c(t) + \sum_{q=1}^N (a_{cq}(o_q(t)) + \Delta M_{cq}(t)) \tilde{h}_q(o_q(t)) \\
 & + \sum_{q=1}^N (b_{cq}(o_q(t)) + \Delta N_{cq}(t)) \tilde{h}_q(o_q(t - \tau)) + P_c(t), \tag{1}
 \end{aligned}$$

where  $0 < \alpha < 1$ ,  $o_c(\cdot)$ , and  $\tilde{h}_c(\cdot)$  indicate the state variables and the corresponding activation functions, respectively.  $a_{cq}(o_q(t))$  and  $b_{cq}(o_q(t))$  signify the connection weights between memristors,  $\tau$  refers to the delay,  $d_c > 0$  is the self-feedback coefficient,  $P_c(t)$  means the external input.  $\Delta M_{cq}(t)$  and  $\Delta N_{cq}(t)$  stand for the uncertain parameters.

By exploiting the differential inclusion theory, system (1) will be rewritten as

$$\begin{aligned}
 {}^C D_t^\alpha o_c(t) \in & -d_c o_c(t) + \sum_{q=1}^N (\text{co}(\underline{a}_{cq}, \bar{a}_{cq}) + \Delta M_{cq}(t)) \tilde{h}_q(o_q(t)) \\
 & + \sum_{q=1}^N (\text{co}(\underline{b}_{cq}, \bar{b}_{cq}) + \Delta N_{cq}(t)) \tilde{h}_q(o_q(t - \tau)) + P_c(t),
 \end{aligned}$$

where

$$a_{cq}(o_q(t)) = \frac{W_{cq}}{C_c} \varepsilon_{cq}, \quad b_{cq}(o_q(t)) = \frac{M_{cq}}{C_c} \varepsilon_{cq}, \quad \varepsilon_{cq} = \begin{cases} 1, & c \neq q \\ -1, & c = q \end{cases}.$$

$W_{cq}$  and  $M_{cq}$  are the resistances of resistors  $P_{cq}$  and  $Q_{cq}$ , respectively.  $P_{cq}$  is the resistors between  $\tilde{h}_q(o_q(t))$  and  $o_c(t)$ .  $Q_{cq}$  is the resistors between  $\tilde{h}_q(o_q(t - \tau))$  and  $o_c(t)$ . Owing to the features of memristor,  $a_{cq}(o_q(t))$  and  $b_{cq}(o_q(t))$  satisfy

$$a_{cq}(o_q(t)) = \begin{cases} a_{cq}^*, & |o_q(t)| < X_q, \\ a_{cq}^{**}, & |o_q(t)| > X_q, \end{cases} \quad \text{and} \quad b_{cq}(o_q(t)) = \begin{cases} b_{cq}^*, & |o_q(t)| < X_q, \\ b_{cq}^{**}, & |o_q(t)| > X_q, \end{cases}$$

where  $a_{cq}^*$ ,  $a_{cq}^{**}$ ,  $b_{cq}^*$ ,  $b_{cq}^{**}$  are constants, the switching jumping  $X_q > 0$ , and

$$\begin{aligned}
 \underline{a}_{cq} &= \min\{a_{cq}^*, a_{cq}^{**}\}, & \bar{a}_{cq} &= \max\{a_{cq}^*, a_{cq}^{**}\}, \\
 \underline{b}_{cq} &= \min\{b_{cq}^*, b_{cq}^{**}\}, & \bar{b}_{cq} &= \max\{b_{cq}^*, b_{cq}^{**}\}, \\
 \mathfrak{A}_{cq}^+ &= \max\{|\underline{a}_{cq}|, |\bar{a}_{cq}|\}, & \mathfrak{B}_{cq}^+ &= \max\{|\underline{b}_{cq}|, |\bar{b}_{cq}|\}.
 \end{aligned}$$

Consequently, there exist  $\mathfrak{A}_{cq}(t) \in \text{co}(\underline{a}_{cq}, \bar{a}_{cq})$  and  $\mathfrak{B}_{cq}(t) \in \text{co}(\underline{b}_{cq}, \bar{b}_{cq})$  such that

$$\text{co}(\underline{a}_{cq}, \bar{a}_{cq}) = \begin{cases} a_{cq}^*, & |o_q(t)| < X_q, \\ \text{co}(\underline{a}_{cq}, \bar{a}_{cq}), & |o_q(t)| = X_q, \\ a_{cq}^{**}, & |o_q(t)| > X_q, \end{cases}$$

and

$$\text{co}(b_{cq}, \bar{b}_{cq}) = \begin{cases} b_{cq}^*, & |o_q(t)| < X_q, \\ \text{co}(\underline{b}_{cq}, \bar{b}_{cq}), & |o_q(t)| = X_q, \\ b_{cq}^{**}, & |o_q(t)| > X_q. \end{cases}$$

Therefore, we can get

$$\begin{aligned} {}_{t_0}^C D_t^\alpha o_c(t) &= -d_c o_c(t) + \sum_{q=1}^N (\mathfrak{A}_{cq}(t) + \Delta M_{cq}(t)) \bar{h}_q(o_q(t)) \\ &\quad + \sum_{q=1}^N (\mathfrak{B}_{cq}(t) + \Delta N_{cq}(t)) \bar{h}_q(o_q(t - \tau)) + P_c(t). \end{aligned} \tag{2}$$

Based on the configuration of system (1), it is possible to derive the subsequent response system:

$$\begin{aligned} {}_{t_0}^C D_t^\alpha s_c(t) &= -d_c s_c(t) + \sum_{q=1}^N (a_{cq}(s_q(t)) + \Delta \bar{M}_{cq}(t)) \bar{h}_q(s_q(t)) \\ &\quad + \sum_{q=1}^N (b_{cq}(s_q(t)) + \Delta \bar{N}_{cq}(t)) \bar{h}_q(s_q(t - \tau)) + P_c(t) + U(t), \end{aligned} \tag{3}$$

where  $U(t)$  is the controller,

$$a_{cq}(s_q(t)) = \begin{cases} a_{cq}^*, & |s_q(t)| < X_q, \\ a_{cq}^{**}, & |s_q(t)| > X_q, \end{cases} \quad \text{and} \quad b_{cq}(s_q(t)) = \begin{cases} b_{cq}^*, & |s_q(t)| < X_q, \\ b_{cq}^{**}, & |s_q(t)| > X_q. \end{cases}$$

The uncertain parameters satisfy  $|\Delta \bar{M}_{cq}(t)| \leq \beth_{cq}$  and  $|\Delta \bar{N}_{cq}(t)| \leq \daleth_{cq}$ , where  $\beth_{cq}$ ,  $\daleth_{cq}$  are positive constants. Hence, system (3) can be reformulated as

$$\begin{aligned} {}_{t_0}^C D_t^\alpha s_c(t) &\in -d_c s_c(t) + \sum_{q=1}^N (\text{co}(\underline{a}_{cq}, \bar{a}_{cq}) + \Delta \bar{M}_{cq}(t)) \bar{h}_q(s_q(t)) \\ &\quad + \sum_{q=1}^N (\text{co}(\underline{b}_{cq}, \bar{b}_{cq}) + \Delta \bar{N}_{cq}(t)) \bar{h}_q(s_q(t - \tau)) + P_c(t) + U_c(t), \end{aligned}$$

where

$$\text{co}(\underline{a}_{cq}, \bar{a}_{cq}) = \begin{cases} a_{cq}^*, & |s_q(t)| < X_q, \\ \text{co}(\underline{a}_{cq}, \bar{a}_{cq}), & |s_q(t)| = X_q, \\ a_{cq}^{**}, & |s_q(t)| > X_q, \end{cases}$$

and

$$\text{co}(\underline{b}_{cq}, \bar{b}_{cq}) = \begin{cases} b_{cq}^*, & |s_q(t)| < X_q, \\ \text{co}(\underline{b}_{cq}, \bar{b}_{cq}), & |s_q(t)| = X_q, \\ b_{cq}^{**}, & |s_q(t)| > X_q. \end{cases}$$

Similarly, there exist  $\bar{\mathfrak{A}}_{cq}(t) \in \text{co}(\underline{a}_{cq}, \bar{a}_{cq})$  and  $\bar{\mathfrak{B}}_{cq}(t) \in \text{co}(\underline{b}_{cq}, \bar{b}_{cq})$  such that

$$\begin{aligned}
 {}^C D_t^\alpha s_c(t) &= -d_c s_c(t) + \sum_{q=1}^N (\bar{\mathfrak{A}}_{cq}(t) + \Delta \bar{M}_{cq}(t)) \bar{h}_q(s_q(t)) \\
 &+ \sum_{q=1}^N (\bar{\mathfrak{B}}_{cq}(t) + \Delta \bar{N}_{cq}(t)) \bar{h}_q(s_q(t - \tau)) + P_c(t) + U(t). \tag{4}
 \end{aligned}$$

Furthermore, the initial conditions for systems (1) and (3) are delineated as follows:

$$o_c(t) = o_c^b(t), \quad s_c(t) = s_c^b(t), \quad t \in [-\tau, 0].$$

**Remark 1.** In [13], investigators stipulated that the activation functions must be bounded. This paper eliminates the requirement for boundedness, making the results more comprehensive and reducing the level of conservatism in the derived synchronization criteria to some degree.

**Remark 2.** In [16, 17, 25], the researchers studied the FTS issues for the integer-order FNNs. In [34], authors conducted an investigation into FTS of FOMNNs in the absence of uncertain parameters. Compared with the results in [16, 17, 25, 34], our research extends the theoretical results from integer-order systems to fractional-order systems, while accounting for the influences of extraneous environmental factors such as time delay and uncertain parameters.

### 4 Main results

The next task is to construct appropriate controllers such that the master-slave systems can realize FTMPs.

Taking the Caputo derivatives of error function  $g_c(t) = s_c(t) - \sum_{k=1}^N \nabla_{ck} o_k(t)$  and utilizing systems (2) and (4), then we can derive the error system as follows:

$$\begin{aligned}
 {}^C D_t^\alpha g_c(t) &= \left[ -d_c s_c(t) + \sum_{q=1}^N (\bar{\mathfrak{A}}_{cq}(t) + \Delta \bar{M}_{cq}(t)) \bar{h}_q(s_q(t)) \right. \\
 &+ \left. \sum_{q=1}^N (\bar{\mathfrak{B}}_{cq}(t) + \Delta \bar{N}_{cq}(t)) \bar{h}_q(s_q(t - \tau)) + P_c(t) + U(t) \right] \\
 &- \sum_{k=1}^N \nabla_{ck} \left[ -d_k o_k(t) + \sum_{q=1}^N (\mathfrak{A}_{kq}(t) + \Delta M_{kq}(t)) \bar{h}_q(o_q(t)) \right. \\
 &+ \left. \sum_{q=1}^t (\mathfrak{B}_{kq}(t) + \Delta N_{kq}(t)) \bar{h}_q(o_q(t - \tau)) + P_k(t) \right].
 \end{aligned}$$

**Remark 3.** The projection factor  $\nabla_{ck}$  considered in this article is no longer a specific constant, but a variable matrix. The continuous change of matrix  $\nabla_{ck}$  will cause the research content of this article to degenerate into the research scope of other similar articles. When the selection matrix  $\nabla_{ck}$  is a positive constant, FTMPs will degenerate into FTCS [24] and degenerate into PS over an infinite time. If  $\nabla_{ck} = 1$ , then the FTMPs will reduce to FTCS [2] and degenerate into CS [33]. If  $\nabla_{ck} = -1$ , then the FTMPs will reduce to FT lag PS [22] and degenerate into lag PS [7]. If  $\nabla_{ck} = \text{diag}(p_1, p_2, \dots, p_c)$ , then the FTMPs will reduce to FT modified PS and degenerate into modified PS.

To facilitate comprehension, we provide the subsequent notations:

$$\begin{aligned} \delta_1 &= \min_{1 < c < \aleph} \left\{ d_c + \omega_c - \sum_{q=1}^{\aleph} (\mathfrak{A}_{qc}^+ + \mathfrak{J}_{qc}) H_c \right\}, & \delta_2 &= \min_{1 < c < \aleph} \left\{ \lambda_c - \sum_{q=1}^{\aleph} (\mathfrak{B}_{qc}^+ + \mathfrak{T}_{qc}) H_c \right\}, \\ \delta'_1 &= 2 \min_{1 < c < \aleph} \left\{ d_c + \omega'_c - \sum_{q=1}^{\aleph} \left( \frac{(\mathfrak{A}_{cq}^+ + \mathfrak{J}_{cq}) H_q}{2} + \frac{(\mathfrak{A}_{qc}^+ + \mathfrak{J}_{qc}) H_c}{2} + \frac{(\mathfrak{B}_{cq}^+ + \mathfrak{T}_{cq}) H_q}{2} \right) \right\}, \\ \delta'_2 &= \max_{1 < c < \aleph} \left\{ \sum_{q=1}^{\aleph} (\mathfrak{B}_{qc}^+ + \mathfrak{T}_{qc}) H_c \right\}, & \delta'_3 &= 2^{1-\theta'} \min_{1 \leq c \leq \aleph} \{ \mu'_c \}, & \delta'_4 &= 2^{1-\xi'} \min_{1 \leq c \leq \aleph} \{ \kappa'_c \}, \\ \delta''_1 &= \min_{1 \leq c \leq \aleph} \left\{ d_c + \omega''_c - \sum_{q=1}^{\aleph} (\mathfrak{A}_{qc}^+) H_c \right\}, & \delta''_2 &= \max_{1 \leq c \leq \aleph} \left\{ \lambda''_c - \sum_{q=1}^{\aleph} (\mathfrak{B}_{qc}^+) H_c \right\}, \\ \delta_3 &= \min_{1 < c < \aleph} \{ \mu_c \}, & \delta_4 &= \min_{1 < c < \aleph} \{ \kappa_c \}, & \delta''_3 &= \min_{1 \leq c \leq \aleph} \{ \mu''_c \}, & \delta''_4 &= \min_{1 \leq c \leq \aleph} \{ \kappa''_c \}. \end{aligned}$$

To complete FTMPs, the time-delay feedback controller is given as

$$U(t) = U_1(t) + U_2(t) + U_3(t) + U_4(t); \tag{51}$$

$$U_1(t) = - \sum_{k=1}^{\aleph} \nabla_{ck} d_k o_k(t) + d_c \sum_{k=1}^{\aleph} \nabla_{ck} o_k(t); \tag{52}$$

$$\begin{aligned} U_2(t) &= \sum_{k=1}^{\aleph} \nabla_{ck} \sum_{q=1}^{\aleph} (\mathfrak{A}_{kq}(t) + \Delta M_{kq}(t)) \mathfrak{h}_q(o_q(t)) \\ &\quad - \sum_{q=1}^{\aleph} (\mathfrak{A}_{cq}(t) + \Delta \bar{M}_{cq}(t)) \mathfrak{h}_q \left( \sum_{k=1}^{\aleph} \nabla_{qk} o_k(t) \right); \end{aligned} \tag{53}$$

$$\begin{aligned} U_3(t) &= \sum_{k=1}^{\aleph} \nabla_{ck} \sum_{q=1}^{\aleph} (\mathfrak{B}_{kq}(t) + \Delta N_{kq}(t)) \mathfrak{h}_q(o_q(t - \tau)) \\ &\quad - \sum_{q=1}^{\aleph} (\mathfrak{B}_{cq}(t) + \Delta \bar{N}_{cq}(t)) \mathfrak{h}_q \left( \sum_{k=1}^{\aleph} \nabla_{qk} o_k(t - \tau) \right); \end{aligned} \tag{54}$$

$$\begin{aligned}
 U_4(t) = & -P_c(t) + \sum_{k=1}^{\aleph} \nabla_{ck} P_k(t) - \omega_c g_c(t) - \lambda_c \text{sign}(g_c(t)) |g_c(t - \tau)| \\
 & - \mu_c \frac{g_c(t)}{[\text{sign}(g_c(t))g_c(t)]^\theta} - \kappa_c \frac{g_c(t)}{[\text{sign}(g_c(t))g_c(t)]^\xi},
 \end{aligned} \tag{5}$$

where  $\omega, \mu_c, \lambda_c, \kappa_c > 0, 0 < \alpha < 1, \theta \geq 1$ , and  $\theta < \xi < 2\theta$ .

**Theorem 1.** *In accordance with Assumption 1 and controller (5), if there exist constants  $\mu_c$  and  $\omega_c$  such that  $\delta_c > 0, c = 1, 2, 3, 4$ , then systems (1) and (3) will actualize FTMPs, and the setting time  $T_1$  satisfies*

$$t^* \leq T_1 = \left[ \frac{\Gamma(1 + \alpha)}{\delta_2 \xi 2^{(\xi - 2\theta)/\theta}} \left( \left( V^\theta(t_0) + \left( \frac{\delta_2}{\delta_1} \right)^{\theta/(\xi - \theta)} \right)^{\xi/\theta} - \left( \frac{\delta_2}{\delta_1} \right)^{\xi/(\xi - \theta)} \right) \right]^{1/\alpha} + t_0.$$

*Proof.* Let us establish the following Lyapunov function:

$$V_1(t) = \sum_{c=1}^{\aleph} |g_c(t)|.$$

From Lemma 1 we can get

$$\begin{aligned}
 {}_{t_0}^C D_t^\alpha V_1(t) &= {}_{t_0}^C D_t^\alpha V_1(t^+) \\
 &\leq \sum_{c=1}^{\aleph} \text{sign}(g_c(t)) {}_{t_0}^C D_t^\alpha g_c(t) \\
 &= \sum_{c=1}^{\aleph} \text{sign}(g_c(t)) \left( -(d_c + \omega_c)g_c(t) - \lambda_c \text{sign}(g_c(t)) |g_c(t - \tau)| \right. \\
 &\quad + \sum_{q=1}^{\aleph} (\bar{\mathfrak{A}}_{cq}(t) + \Delta \bar{M}_{cq}(t)) \left[ \hbar_q(s_q(t)) - \hbar_q \left( \sum_{k=1}^{\aleph} \nabla_{qk} o_k(t) \right) \right] \\
 &\quad + \sum_{q=1}^{\aleph} (\bar{\mathfrak{B}}_{cq}(t) + \Delta \bar{N}_{cq}(t)) \left[ \hbar_q(s_q(t - \tau)) - \hbar_q \left( \sum_{k=1}^{\aleph} \nabla_{qk} o_k(t - \tau) \right) \right] \\
 &\quad \left. - \mu_c \frac{g_c(t)}{[\text{sign}(g_c(t))g_c(t)]^\theta} - \kappa_c \frac{g_c(t)}{[\text{sign}(g_c(t))g_c(t)]^\xi} \right).
 \end{aligned} \tag{6}$$

Based on Assumption 1, one has

$$\begin{aligned}
 &\sum_{c=1}^{\aleph} \text{sign}(g_c(t)) \left[ \sum_{q=1}^{\aleph} (\bar{\mathfrak{A}}_{cq}(t) + \Delta \bar{M}_{cq}(t)) \left( \hbar_q(s_q(t)) - \hbar_q \left( \sum_{k=1}^{\aleph} \nabla_{qk} o_k(t) \right) \right) \right] \\
 &\leq \sum_{c=1}^{\aleph} \sum_{q=1}^{\aleph} (\mathfrak{A}_{cq}^+ + \mathfrak{B}_{cq}) H_q |g_q(t)|.
 \end{aligned} \tag{7}$$

Similarly, we get

$$\sum_{c=1}^{\aleph} \text{sign}(g_c(t)) \left[ \sum_{q=1}^{\aleph} (\mathfrak{B}_{cq}(t) + \Delta \bar{N}_{cq}(t)) \left( \hbar_q(s_q(t-\tau)) - \hbar_q \left( \sum_{k=1}^{\aleph} \nabla_{qk} o_k(t-\tau) \right) \right) \right] \leq \sum_{c=1}^{\aleph} \sum_{q=1}^{\aleph} (\mathfrak{B}_{cq}^+ + \Upsilon_{cq}) H_q |g_q(t-\tau)|. \tag{8}$$

Bringing inequalities (7) and (8) into (6), we have

$$\begin{aligned} {}_{t_0}^C D_t^\alpha V_1(t) &\leq \sum_{c=1}^{\aleph} \left( -(d_c + \omega_c) |g_c(t)| + \sum_{q=1}^{\aleph} (\mathfrak{A}_{cq}^+ + \mathfrak{J}_{cq}) H_q |g_q(t)| \right. \\ &\quad \left. + \sum_{q=1}^{\aleph} (\mathfrak{B}_{cq}^+ + \Upsilon_{cq}) H_q |g_q(t-\tau)| - \lambda_c |g_c(t-\tau)| \right. \\ &\quad \left. - \mu_c |g_c(t)|^{-(\theta-1)} - \kappa_c |g_c(t)|^{-(\xi-1)} \right) \\ &= - \sum_{c=1}^{\aleph} \left( d_c + \omega_c - \sum_{q=1}^{\aleph} (\mathfrak{A}_{qc}^+ + \mathfrak{J}_{qc}) H_c \right) |g_c(t)| \\ &\quad - \sum_{c=1}^{\aleph} \left( \lambda_c - \sum_{q=1}^{\aleph} (\mathfrak{B}_{qc}^+ + \Upsilon_{qc}) H_c \right) |g_c(t-\tau)| \\ &\quad - \sum_{c=1}^{\aleph} \mu_c |g_c(t)|^{-(\theta-1)} - \sum_{c=1}^{\aleph} \kappa_c |g_c(t)|^{-(\xi-1)} \\ &\leq -\delta_1 V_1^{-(\theta-1)}(t) - \delta_2 V_1^{-(\xi-1)}(t). \end{aligned}$$

In accordance with Lemma 4, systems (1) and (3) can realize FTMPs. □

Next, we will consider the synchronization case in the 2-norm case to achieve a more comprehensive outcome. The nonlinear feedback controller has been restructured in the following manner:

$$U(t) = U_1(t) + U_2(t) + U_3(t) + U_4(t); \tag{9_1}$$

$$U_1(t) = - \sum_{k=1}^{\aleph} \nabla_{ck} d_k o_k(t) + d_c \sum_{k=1}^{\aleph} \nabla_{ck} o_k(t); \tag{9_2}$$

$$\begin{aligned} U_2(t) &= \sum_{k=1}^{\aleph} \nabla_{ck} \sum_{q=1}^{\aleph} (\mathfrak{A}_{kq}(t) + \Delta M_{kq}(t)) \hbar_q(o_q(t)) \\ &\quad - \sum_{q=1}^{\aleph} (\mathfrak{A}_{cq}(t) + \Delta \bar{M}_{cq}(t)) \hbar_q \left( \sum_{k=1}^{\aleph} \nabla_{qk} o_k(t) \right); \end{aligned} \tag{9_3}$$

$$\begin{aligned}
 U_3(t) = & \sum_{k=1}^{\aleph} \nabla_{ck} \sum_{q=1}^{\aleph} (\mathfrak{B}_{kq}(t) + \Delta N_{kq}(t)) \hbar_q(o_q(t - \tau)) \\
 & - \sum_{q=1}^{\aleph} (\overline{\mathfrak{B}}_{cq}(t) + \Delta \overline{N}_{cq}(t)) \hbar_q \left( \sum_{k=1}^{\aleph} \nabla_{qk} o_k(t - \tau) \right); \tag{94}
 \end{aligned}$$

$$\begin{aligned}
 U_4(t) = & -P_c(t) + \sum_{k=1}^{\aleph} \nabla_{ck} P_k(t) - \mu'_c \frac{g_c(t)}{[g_c(t)^2]^{\theta'}} - \kappa'_c \frac{g_c(t)}{[g_c(t)^2]^{\xi'}} \\
 & - \omega'_c g_c(t), \tag{95}
 \end{aligned}$$

where  $\mu'_c, \kappa'_c, \omega'_c > 0, 0 < \alpha < 1, \theta' \geq 1$ , and  $\theta' < \xi' < 2\theta'$ .

**Theorem 2.** According to Assumption 1 and controller (9), if there exist constants  $w, \mu'_c$ , and  $\omega'_c$  such that

$$\delta'_c > 0, \quad c = 1, 2, 3, 4, \quad \delta_1 - w\delta'_2 > 0,$$

then systems (1) and (3) can reach FTMPs. The settling time  $T_2$  is expressed as

$$\begin{aligned}
 t^* & \leq T_2 \\
 & = \left[ \frac{\Gamma(1 + \alpha)}{\delta_4 \xi'^{2(\xi' - 2\theta')/\theta'}} \left( \left( V^{\theta'}(t_0) + \left( \frac{\delta_4}{\delta_3} \right)^{\theta' / (\xi' - \theta')} \right)^{\xi' / \theta'} - \left( \frac{\delta_4}{\delta_2} \right)^{\xi' / (\xi' - \theta')} \right) \right]^{1/\alpha} + t_0.
 \end{aligned}$$

*Proof.* Let us consider the following Lyapunov function:

$$V_2(t) = \frac{1}{2} \sum_{c=1}^{\aleph} g_c^2(t).$$

In the light of Lemma 2, we have

$$\begin{aligned}
 C_{t_0} D_t^\alpha V_2(t) & = C_{t_0} D_t^\alpha \left( \frac{1}{2} \sum_{c=1}^{\aleph} g_c^2(t) \right) \leq \sum_{c=1}^{\aleph} g_c(t) {}_{t_0} D_t^\alpha g_c(t) \\
 & = \sum_{c=1}^{\aleph} g_c(t) \left\{ \left[ -d_c s_c(t) + \sum_{q=1}^{\aleph} (\overline{\mathfrak{A}}_{qc}(t) + \Delta \overline{M}_{cq}(t)) \hbar_q(s_q(t)) \right. \right. \\
 & \quad \left. \left. + \sum_{q=1}^{\aleph} (\overline{\mathfrak{B}}_{cq}(t) + \Delta \overline{N}_{cq}(t)) \hbar_q(s_q(t - \tau)) + P_c(t) + U(t) \right] \right. \\
 & \quad \left. - \sum_{k=1}^{\aleph} \nabla_{ck} \left[ -d_k x_k(t) + \sum_{q=1}^{\aleph} (\mathfrak{A}_{kq}(t) + \Delta M_{kq}(t)) \hbar_q(o_q(t)) \right. \right. \\
 & \quad \left. \left. + \sum_{q=1}^{\aleph} (\mathfrak{B}_{kq}(t) + \Delta N_{kq}(t)) \hbar_q(o_q(t - \tau)) + P_k(t) \right] \right\}
 \end{aligned}$$

$$\begin{aligned}
 &= \sum_{c=1}^N g_c(t) \left[ -(d_c + \omega'_c)g_c(t) - \mu'_c g_c^{2-2\theta'}(t) - \kappa'_c g_c^{2-2\xi'}(t) \right. \\
 &\quad + \sum_{q=1}^N (\overline{\mathfrak{A}}_{cq}(t) + \Delta \overline{M}_{cq}(t)) \left( \hbar_q(s_q(t)) - r_q \left( \sum_{k=1}^N \nabla_{qk} o_k(t) \right) \right) \\
 &\quad \left. + \sum_{q=1}^N (\overline{\mathfrak{B}}_{cq}(t) + \Delta \overline{N}_{cq}(t)) \left( \hbar_q(s_q(t-\tau)) - r_q \left( \sum_{k=1}^N \nabla_{qk} o_k(t-\tau) \right) \right) \right].
 \end{aligned}$$

Under conditions of Assumption 1, memristors, and uncertain parameters, we can get

$$\begin{aligned}
 {}^C_{t_0}D_t^\alpha V_2(t) &\leq \sum_{c=1}^N -(d_c + \omega'_c)g_c^2(t) + \sum_{c=1}^N \sum_{q=1}^N \frac{(\mathfrak{A}_{cq}^+ + \mathfrak{J}_{cq})H_q}{2} (g_c^2(t) + g_q^2(t)) \\
 &\quad + \sum_{c=1}^N \sum_{q=1}^N \frac{(\mathfrak{B}_{cq}^+ + \mathfrak{I}_{cq})H_q}{2} (g_c^2(t) + g_q^2(t-\tau)) \\
 &\quad - \sum_{c=1}^N \mu'_c (g_c^2(t))^{1-\theta'} - \sum_{c=1}^N \kappa'_c (g_c^2(t))^{1-\xi'}.
 \end{aligned}$$

Consequently, we have

$$\begin{aligned}
 {}^C_{t_0}D_t^\alpha V_2(t) &\leq -2 \sum_{c=1}^N \left( d_c + \omega'_c - \sum_{q=1}^N \frac{(\mathfrak{A}_{cq}^+ + \mathfrak{J}_{cq})H_q}{2} \right. \\
 &\quad \left. - \sum_{q=1}^N \frac{(\mathfrak{A}_{qc}^+ + \mathfrak{J}_{qc})H_c}{2} - \sum_{q=1}^N \frac{(\mathfrak{B}_{cq}^+ + \mathfrak{I}_{cq})H_q}{2} \right) \frac{1}{2} g_c^2(t) \\
 &\quad + \sum_{c=1}^N \sum_{q=1}^N (\mathfrak{B}_{qc}^+ + \mathfrak{I}_{qc}) H_c \frac{1}{2} g_c^2(t-\tau) \\
 &\quad - 2^{1-\theta'} \mu'_c \left( \frac{1}{2} \sum_{c=1}^N g_c^2(t) \right)^{1-\theta'} - 2^{1-\xi'} \kappa'_c \left( \frac{1}{2} \sum_{c=1}^N g_c^2(t) \right)^{1-\xi'} \\
 &\leq -\delta'_1 V_2(t) + \delta'_2 V_2(t-\tau) - \delta'_3 V_3^{1-\theta'}(t) - \delta'_4 V_2^{1-\xi'}(t) \\
 &\leq -(\delta'_1 - w\delta'_2) V_2(t) - \delta'_3 V_3^{-(\theta'-1)}(t) - \delta'_4 V_2^{-(\xi'-1)}(t) \\
 &\leq -\delta'_3 V_3^{-(\theta'-1)}(t) - \delta'_4 V_2^{-(\xi'-1)}(t)
 \end{aligned}$$

for some  $w > 1$ . From Lemma 4 it is shown that systems (1) and (3) can realize FTMPS. □

Particularly, if  $\Delta M_{cq}(t) = \Delta N_{cq}(t) = \Delta \overline{M}_{cq}(t) = \Delta \overline{N}_{cq}(t) = 0$ , then systems (1) and (3) will degenerate into FOMDNNs without uncertain parameters. The master system

is defined as follows:

$$\begin{aligned}
 {}^C_{t_0}D_t^\alpha o_c(t) &= -d_c o_c(t) + \sum_{q=1}^N (a_{cq}(o_q(t))) \bar{h}_q(o_q(t)) \\
 &\quad + \sum_{q=1}^N (b_{cq}(o_q(t))) \bar{h}_q(o_q(t - \tau)) + P_c(t),
 \end{aligned} \tag{10}$$

and the slave system is

$$\begin{aligned}
 {}^C_{t_0}D_t^\alpha s_c(t) &= -d_c s_c(t) + \sum_{q=1}^N (a_{cq}(s_q(t))) \bar{h}_q(s_q(t)) \\
 &\quad + \sum_{q=1}^N (b_{cq}(s_q(t))) \bar{h}_q(s_q(t - \tau)) + P_c(t) + U(t).
 \end{aligned} \tag{11}$$

Let the controller be designed as

$$U(t) = U_1(t) + U_2(t) + U_3(t) + U_4(t); \tag{121}$$

$$U_1(t) = - \sum_{k=1}^N \nabla_{ck} d_k o_k(t) + d_c \sum_{k=1}^N \nabla_{ck} o_k(t); \tag{122}$$

$$U_2(t) = \sum_{k=1}^N \nabla_{ck} \sum_{q=1}^N (\mathfrak{A}_{kq}(t)) \bar{h}_q(o_q(t)) - \sum_{q=1}^N (\bar{\mathfrak{A}}_{cq}(t)) \bar{h}_q \left( \sum_{k=1}^N \nabla_{qk} o_k(t) \right); \tag{123}$$

$$\begin{aligned}
 U_3(t) &= \sum_{k=1}^N \nabla_{ck} \sum_{q=1}^N (\mathfrak{B}_{kq}(t)) \bar{h}_q(o_q(t - \tau)) \\
 &\quad - \sum_{q=1}^N (\bar{\mathfrak{B}}_{cq}(t)) \bar{h}_q \left( \sum_{k=1}^N \nabla_{qk} o_k(t - \tau) \right);
 \end{aligned} \tag{124}$$

$$\begin{aligned}
 U_4(t) &= -P_c(t) + \sum_{k=1}^N \nabla_{ck} P_k(t) - \omega_c'' g_c(t) - \lambda_c'' \text{sign}(g_c(t)) |g_c(t - \tau)| \\
 &\quad - \mu_c'' \frac{g_c(t)}{[\text{sign}(g_c(t))g_c(t)]^{\theta''}} - \kappa_c'' \frac{g_c(t)}{[\text{sign}(g_c(t))g_c(t)]^{\xi''}},
 \end{aligned} \tag{125}$$

where  $\omega_c'', \lambda_c'', \mu_c'', \kappa_c'' > 0$ ,  $\theta'' \geq 1$ , and  $\theta'' < \xi'' < 2\theta''$ . Then Corollary 1 holds.

**Corollary 1.** *Based on Assumption 1 and controller (12), if there exist constants  $\mu_c''$  and  $\omega_c''$  such that  $\delta_c'' > 0$ ,  $c = 1, 2, 3, 4$ , then systems (10) and (11) can achieve the FTMPs.*

*Proof.* The proof of the corollary closely resembles that of Theorem 1. Therefore, it has been omitted. □

**Remark 4.** In Theorem 1, the settling time  $t_1$  is affected by parameter  $\alpha$ ,  $\theta$ ,  $\omega_c$ , and  $\mu_c$ . We can modify the control parameters to ensure that the settling time  $t_1$  aligns with practical requirements. Obviously, the larger the parameter  $\mu_c$ , the smaller the resulting settling time, thus the control parameter  $\mu_c$  should as larger as possible in the real situation. Besides, the lower the starting value (corresponding to  $V_1(t_0)$ ) of the system, the shorter the time  $t_1$ . A comparable argument can be made for the settling time  $t_2$  outlined in Theorem 2.

### 5 Numerical simulations

To evaluate the accuracy of the findings, two numerical illustrations will be presented for verification.

*Example 1.* Let us consider the drive system

$$\begin{aligned}
 {}^C D_t^\alpha o_1(t) &= -d_1 o_1(t) + \sum_{q=1}^2 (a_{1q} + \Delta M_{1q}) \tanh(o_q(t)) \\
 &\quad + \sum_{q=1}^2 (b_{1q} + \Delta N_{1q}) \tanh(o_q(t - \tau)) + P_1(t)
 \end{aligned}$$

and

$$\begin{aligned}
 {}^C D_t^\alpha o_2(t) &= -d_2 o_2(t) + \sum_{q=1}^2 (a_{2q} + \Delta M_{2q}) \tanh(o_q(t)) \\
 &\quad + \sum_{q=1}^2 (b_{2q} + \Delta N_{2q}) \tanh(o_q(t - \tau)) + P_2(t), \tag{13}
 \end{aligned}$$

where  $\alpha = 0.98$ ,  $\tau = 1$ ,  $d_1 = 0.45$ ,  $d_2 = 0.4$ ,  $P_1(t) = P_2(t) = 0$ . Define the parameters of the memristor as follows:

$$\begin{aligned}
 a_{11} &= \begin{cases} 0.27, & |o_1(t)| \leq 1, \\ -0.3, & |o_1(t)| > 1, \end{cases} & a_{12} &= \begin{cases} -0.2, & |o_2(t)| \leq 1, \\ -0.3, & |o_2(t)| > 1, \end{cases} \\
 a_{21} &= \begin{cases} 0.75, & |o_1(t)| \leq 1, \\ 0.7, & |o_1(t)| > 1, \end{cases} & a_{22} &= \begin{cases} -0.2, & |o_2(t)| \leq 1, \\ -0.1, & |o_2(t)| > 1, \end{cases} \\
 b_{11} &= \begin{cases} -0.05, & |o_1(t)| < 1, \\ -0.01, & |o_1(t)| > 1, \end{cases} & b_{12} &= \begin{cases} -0.4, & |o_2(t)| \leq 1, \\ -0.2, & |o_2(t)| > 1, \end{cases} \\
 b_{21} &= \begin{cases} 0.25, & |o_1(t)| \leq 1, \\ 0.6, & |o_1(t)| > 1, \end{cases} & b_{22} &= \begin{cases} 0.3, & |o_2(t)| \leq 1, \\ 0.2, & |o_2(t)| > 1. \end{cases}
 \end{aligned}$$

The uncertain parameters are selected as

$$\Delta M_{cq}(t) = \begin{pmatrix} 0.2 \sin t & 0.1 \cos t \\ 0.1 \cos t & 0.2 \sin t \end{pmatrix} \quad \text{and} \quad \Delta N_{cq} = \begin{pmatrix} 0.1 \sin t & 0.3 \sin t \\ 0.3 \cos t & 0.2 \cos t \end{pmatrix}.$$

By simple calculation, we can get  $\mathfrak{A}_{11}^+ = 0.3, \mathfrak{A}_{12}^+ = 0.3, \mathfrak{A}_{21}^+ = 0.75, \mathfrak{A}_{22}^+ = 0.2, \mathfrak{B}_{11}^+ = 0.05, \mathfrak{B}_{12}^+ = 0.4, \mathfrak{B}_{21}^+ = 0.6, \mathfrak{B}_{22}^+ = 0.3$ . The response systems are

$$\begin{aligned} {}^C D_t^\alpha s_1(t) &= -d_1 s_1(t) + \sum_{q=1}^2 (a_{1q} + \Delta \bar{M}_{1q}) \tanh(s_q(t)) \\ &\quad + \sum_{q=1}^2 (b_{1q} + \Delta \bar{N}_{1q}) \tanh(s_q(t - \tau)) + P_1(t) + U(t) \end{aligned}$$

and

$$\begin{aligned} {}^C D_t^\alpha s_2(t) &= -d_2 s_2(t) + \sum_{q=1}^2 (a_{2q} + \Delta \bar{M}_{2q}) \tanh(s_q(t)) \\ &\quad + \sum_{q=1}^2 (b_{2q} + \Delta \bar{N}_{2q}) \tanh(s_q(t - \tau)) + P_2(t) + U(t), \end{aligned} \tag{14}$$

where

$$\begin{aligned} a_{11} &= \begin{cases} 0.27, & |s_1(t)| \leq 1, \\ -0.3, & |s_1(t)| > 1, \end{cases} & a_{12} &= \begin{cases} -0.2, & |s_2(t)| \leq 1, \\ -0.3, & |s_2(t)| > 1, \end{cases} \\ a_{21} &= \begin{cases} 0.75, & |s_1(t)| \leq 1, \\ 0.7, & |s_1(t)| > 1, \end{cases} & a_{22} &= \begin{cases} -0.2, & |s_2(t)| \leq 1, \\ -0.1, & |s_2(t)| > 1, \end{cases} \\ b_{11} &= \begin{cases} -0.05, & |s_1(t)| \leq 1, \\ -0.01, & |s_1(t)| > 1, \end{cases} & b_{12} &= \begin{cases} -0.4, & |s_2(t)| \leq 1, \\ -0.2, & |s_2(t)| > 1, \end{cases} \\ b_{21} &= \begin{cases} 0.25, & |s_1(t)| \leq 1, \\ 0.6, & |s_1(t)| > 1, \end{cases} & b_{22} &= \begin{cases} 0.3, & |s_2(t)| \leq 1, \\ 0.2, & |s_2(t)| > 1. \end{cases} \end{aligned}$$

The uncertain parameters are selected as

$$\Delta \bar{M}_{cq}(t) = \begin{pmatrix} 0.1 \cos t & 0.1 \sin t \\ 0.2 \sin t & 0.2 \cos t \end{pmatrix} \quad \text{and} \quad \Delta \bar{N}_{cq}(t) = \begin{pmatrix} 0.3 \sin t & 0.3 \sin t \\ 0.1 \cos t & 0.2 \sin t \end{pmatrix}.$$

Then we get  $\mathfrak{J}_{11} = 0.1, \mathfrak{J}_{12} = 0.1, \mathfrak{J}_{21} = 0.2, \mathfrak{J}_{22} = 0.2, \mathfrak{T}_{11} = 0.3, \mathfrak{T}_{12} = 0.3, \mathfrak{T}_{21} = 0.1, \mathfrak{T}_{22} = 0.2$ . The remaining parameters are consistent with system (13). The trajectories of systems (13) and (14) in the absence of controller are depicted in Figs. 1–2.

Selecting the projective matrix  $\nabla_{ck}$  as

$$\nabla_{ck} = \nabla_{2 \times 2} = \begin{pmatrix} 2 & -0.3 \\ -0.4 & 1.2 \end{pmatrix},$$

the error functions of MPS are computed as

$$\begin{aligned} g_1 &= s_1 - \sum_{k=1}^2 \nabla_{1k} o_k = s_1 - (2o_1 - 0.3o_2), \\ g_2 &= s_2 - \sum_{k=1}^2 \nabla_{2k} o_k = s_2 - (-0.4o_1 + 1.2o_2). \end{aligned} \tag{15}$$

To attain the FTMPS of systems (13) and (14), the parameters  $\omega_c, \theta, \mu_c, \xi, \kappa_c, \lambda_c$  in controller (5) are chosen as  $\omega_1 = 13, \omega_2 = 12.2, \theta = 1, \mu_1 = \mu_2 = 2, \xi = 1.1, \kappa_1 = \kappa_2 = 1, \lambda_1 = 3.5, \lambda_2 = 4.5$ . We can easily get  $\delta_1 = 11.8 > 0, \delta_2 = 2.45 > 0, \delta_3 = 2, \text{ and } \delta_4 = 1$ . Therefore, systems (13) and (14) can realize the FTMPS. In addition, the settling time is  $T_1 \approx 2.0878$ .

By observing Figs. 1 and 2, we can know that  $o_1$  and  $s_1, o_2$  and  $s_2$  cannot be synchronized in the absence of controllers for systems (13) and (14). After adding controller (5), systems (13) and (14) can implement FTMPS and are shown in Figs. 3 and 4. Furthermore, Fig. 5 illustrates that the temporal trends of errors  $g_1(t)$  and  $g_2(t)$ , as outlined in Eq. (15), exhibit rapid decay towards zero as time approaches infinity, which revealed that systems (13) and (14) can realize FTMPS in the case of controller (5). Figure 6 shows the trajectory of the error functions  $g_c(t)$  in the 1-norm.

*Example 2.* Firstly, we choose the two-dimensional FOMDNNs systems

$$\begin{aligned}
 {}^C D_t^\alpha o_c(t) &= -d_c o_c(t) + \sum_{q=1}^2 (a_{cq}(o_c(t)) + \Delta M_{cq}(t)) r_q(o_q(t)) \\
 &+ \sum_{q=1}^2 (b_{cq}(o_c(t)) + \Delta N_{cq}(t)) r_q(o_q(t - \tau)) + P_c(t)
 \end{aligned} \tag{16}$$

and

$$\begin{aligned}
 {}^C D_t^\alpha s_c(t) &= -d_c s_c(t) + \sum_{q=1}^2 (a_{cq}(s_c(t)) + \Delta \bar{M}_{cq}(t)) r_q(s_q(t)) \\
 &+ \sum_{q=1}^2 (b_{cq}(s_c(t)) + \Delta \bar{N}_{cq}(t)) r_q(s_q(t - \tau)) + P_c(t) + U(t),
 \end{aligned} \tag{17}$$

where  $\alpha = 0.95, \tau = 1, d_1 = 0.3, \text{ and } d_2 = 0.5$ . Besides,

$$\begin{aligned}
 a_{11} &= \begin{cases} -0.1, & |o_1(t)| \leq 1, \\ 0.1, & |o_1(t)| > 1, \end{cases} & a_{12} &= \begin{cases} -0.1, & |o_2(t)| \leq 1, \\ 0.05, & |o_2(t)| > 1, \end{cases} \\
 a_{21} &= \begin{cases} 0.3, & |o_1(t)| \leq 1, \\ -0.3, & |o_1(t)| > 1, \end{cases} & a_{22} &= \begin{cases} 0.15, & |o_2(t)| \leq 1, \\ -0.15, & |o_2(t)| > 1, \end{cases} \\
 b_{11} &= \begin{cases} -0.1, & |o_1(t)| \leq 1, \\ 0.1, & |o_1(t)| > 1, \end{cases} & b_{12} &= \begin{cases} -0.85, & |o_2(t)| \leq 1, \\ -0.3, & |o_2(t)| > 1, \end{cases} \\
 b_{21} &= \begin{cases} 0.3, & |o_1(t)| \leq 1, \\ -0.3, & |o_1(t)| > 1, \end{cases} & b_{22} &= \begin{cases} -0.2, & |o_2(t)| \leq 1, \\ 0.2, & |o_2(t)| > 1. \end{cases}
 \end{aligned}$$

The uncertain parameters  $\Delta M_{cq}(t)$  and  $\Delta N_{cq}(t)$  are selected as

$$\Delta M_{cq}(t) = \begin{pmatrix} 0.2 \sin t & 0.1 \cos t \\ 0.1 \cos t & 0.2 \sin t \end{pmatrix} \quad \text{and} \quad \Delta N_{cq} = \begin{pmatrix} 0.1 \sin t & 0.3 \sin t \\ 0.3 \cos t & 0.2 \cos t \end{pmatrix}.$$

Similarly, we consider corresponding parameters of system (17) as

$$\begin{aligned}
 a_{11} &= \begin{cases} -0.1, & |s_1(t)| \leq 1, \\ 0.1, & |s_1(t)| > 1, \end{cases} & a_{12} &= \begin{cases} -0.1, & |s_2(t)| \leq 1, \\ 0.05, & |s_2(t)| > 1, \end{cases} \\
 a_{21} &= \begin{cases} 0.3, & |s_1(t)| \leq 1, \\ -0.3, & |s_1(t)| > 1, \end{cases} & a_{22} &= \begin{cases} 0.15, & |s_2(t)| \leq 1, \\ -0.15, & |s_2(t)| > 1, \end{cases} \\
 b_{11} &= \begin{cases} -0.1, & |s_1(t)| \leq 1, \\ 0.1, & |s_1(t)| > 1, \end{cases} & b_{12} &= \begin{cases} -0.85, & |s_2(t)| \leq 1, \\ -0.3, & |s_2(t)| > 1, \end{cases} \\
 b_{21} &= \begin{cases} 0.3, & |s_1(t)| \leq 1, \\ -0.3, & |s_1(t)| > 1, \end{cases} & b_{22} &= \begin{cases} -0.2, & |s_2(t)| \leq 1, \\ 0.2, & |s_2(t)| > 1. \end{cases}
 \end{aligned}$$

The uncertain parameters  $\Delta\bar{M}_{cq}(t)$  and  $\Delta\bar{N}_{cq}(t)$  are selected as

$$\Delta\bar{M}_{cq}(t) = \begin{pmatrix} 0.1 \cos t & 0.1 \sin t \\ 0.2 \sin t & 0.2 \cos t \end{pmatrix} \quad \text{and} \quad \Delta\bar{N}_{cq} = \begin{pmatrix} 0.3 \sin t & 0.3 \sin t \\ 0.1 \cos t & 0.2 \sin t \end{pmatrix}.$$

By simple calculation, we can get  $\mathfrak{A}_{11}^+ = 0.1, \mathfrak{A}_{12}^+ = 0.1, \mathfrak{A}_{21}^+ = 0.3, \mathfrak{A}_{22}^+ = 0.15, \mathfrak{B}_{11}^+ = 0.1, \mathfrak{B}_{12}^+ = 0.85, \mathfrak{B}_{21}^+ = 0.3, \mathfrak{B}_{22}^+ = 0.2, \mathfrak{C}_{11} = 0.1, \mathfrak{C}_{12} = 0.1, \mathfrak{C}_{21} = 0.2, \mathfrak{C}_{22} = 0.2, \mathfrak{T}_{11} = 0.3, \mathfrak{T}_{12} = 0.3, \mathfrak{T}_{21} = 0.1, \mathfrak{T}_{22} = 0.2$ . Selecting the projective matrix  $\nabla_{ck}$  as

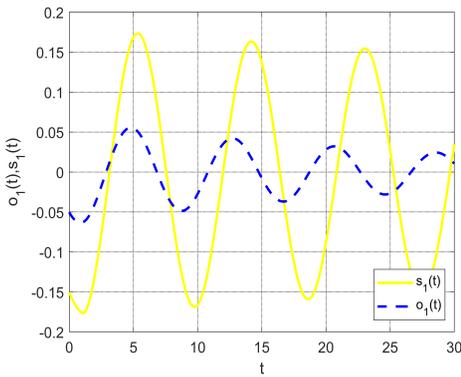
$$\nabla_{ck} = \nabla_{2 \times 2} = \begin{pmatrix} 1.6 & -1.5 \\ 0.1 & 0.2 \end{pmatrix},$$

the error functions of FTMPs are computed as

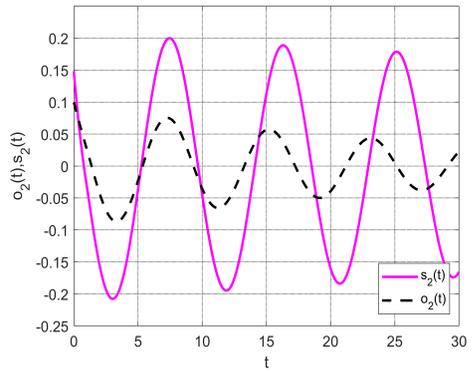
$$\begin{aligned}
 g_1 &= s_1 - \sum_{k=1}^2 \nabla_{1k} o_k = s_1 - (1.6o_1 - 1.5o_2), \\
 g_2 &= s_2 - \sum_{k=1}^2 \nabla_{2k} o_k = s_2 - (0.1o_1 + 0.2o_2).
 \end{aligned} \tag{18}$$

In order to fulfill the FTMPs of systems (16) and (17), the parameters  $\omega'_c, \theta', \mu'_c, \kappa'_c, \xi'$  in controller (9) are chosen as  $\omega'_1 = \omega'_2 = 15, \theta' = 1, \xi' = 1.5, \mu'_1 = \mu'_2 = 2$ , and  $\kappa'_1 = \kappa'_2 = 2$ . We can easily get  $\delta'_1 = 27.95 > 0, \delta'_2 = 1.35 > 0, \delta'_3 = 2, \delta'_4 \approx 1.414$ . Therefore, systems (16) and (17) can realize the FTMPs. In addition, the settling time is  $T_2 \approx 1.115$ .

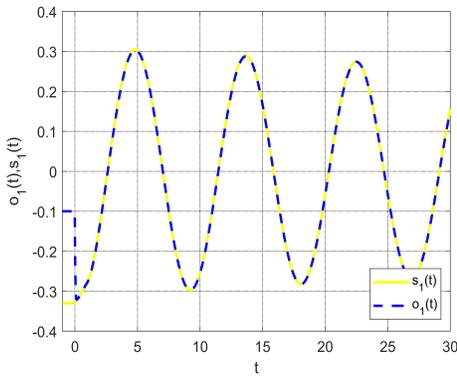
By observing Figs. 7 and 8, we can know that  $o_1$  and  $s_1, o_2$  and  $s_2$  cannot be synchronized in the absence of controllers. After adding controller (9), systems (16) and (17) can implement FTMPs, as shown in Figs. 9 and 10. Furthermore, Fig. 11 illustrates that the temporal trends of errors  $g_1(t)$  and  $g_2(t)$ , as outlined in Eq. (18), exhibit rapid decay towards zero as time approaches infinity, which revealed that systems (16) and (17) can realize FTMPs in the case of controller (9). Figure 12 illustrates the path of the error function  $g_c(t)$  in the 2-norm.



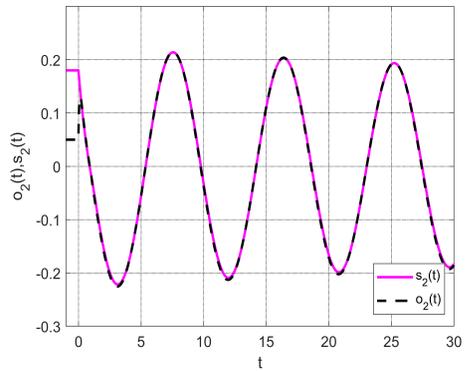
**Figure 1.** The state trajectories of systems (13) and (14) without controllers.



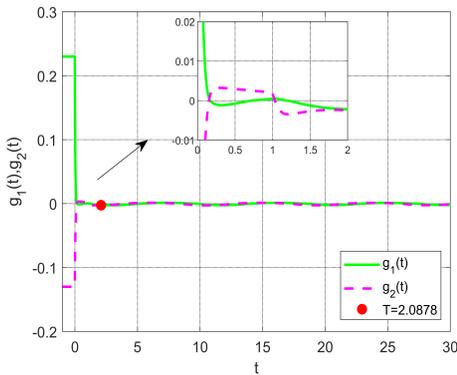
**Figure 2.** The state trajectories of systems (13) and (14) without controllers.



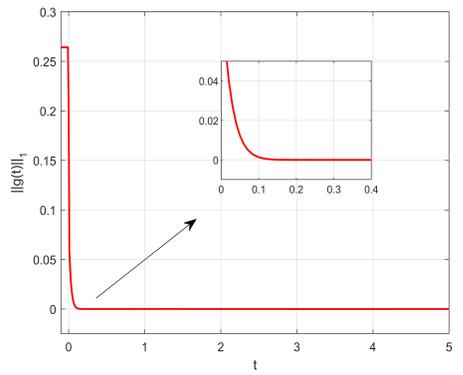
**Figure 3.** The state trajectories of systems (13) and (14) under controller (5).



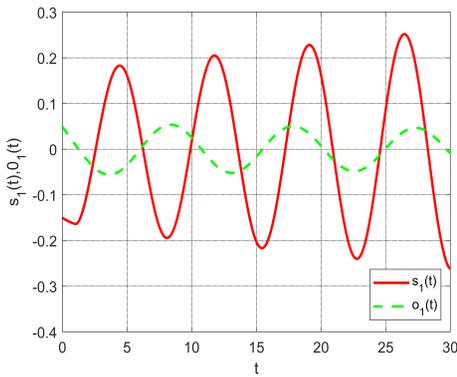
**Figure 4.** The state trajectories of systems (13) and (14) under controller (5).



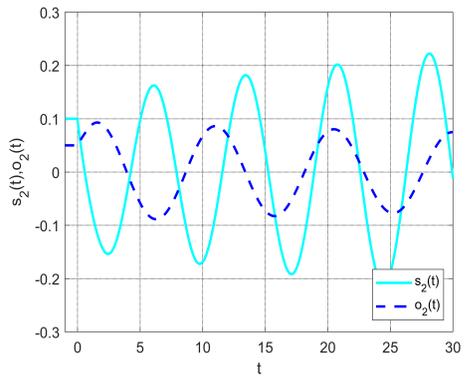
**Figure 5.** The trajectories of error  $g_1$  and  $g_2$ .



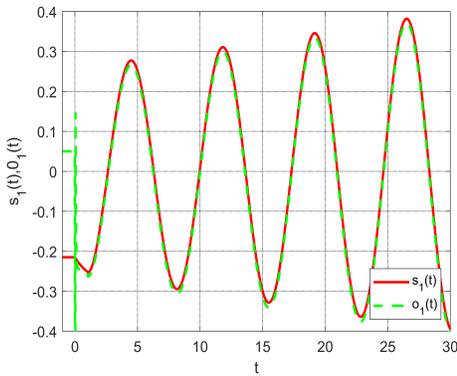
**Figure 6.** The trajectory of  $\|g(t)\|_1$ .



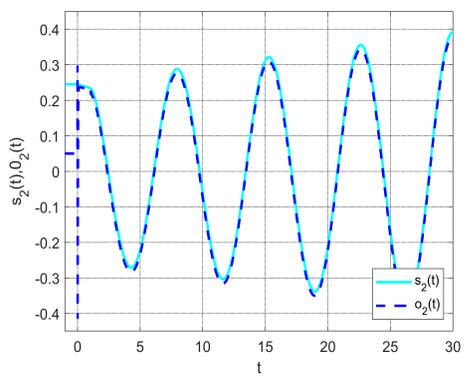
**Figure 7.** The state trajectories of systems (16) and (17) without controllers.



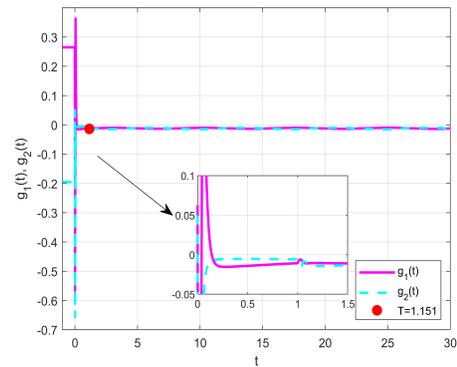
**Figure 8.** The state trajectories of systems (16) and (17) without controllers.



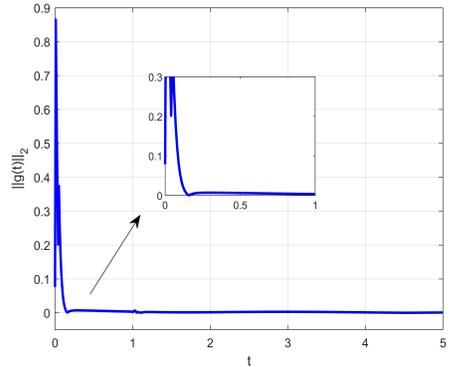
**Figure 9.** The state trajectories of systems (16) and (17) under controller (9).



**Figure 10.** The state trajectories of systems (16) and (17) under controller (9).



**Figure 11.** The trajectories of error  $g_1$  and  $g_2$ .



**Figure 12.** The trajectory of  $\|g(t)\|_2$ .

## 6 Conclusion

This article examines the issues associated with FTMPs for a class of FODMNNs. We extend the common infinite-time interval and constant projective scaling factor to finite-time synchronization of constant matrix projective scaling factor. The sign function is applied to the controller, and it is proved that the selected systems can actualize FTMPs under different norm cases. The accuracy of Theorems 1 and 2 is verified through the examination of two numerical illustrations. Event-triggered control has been proven to be an effective strategy in conserving communication resources, reducing energy consumption in control processes and enhancing system robustness. These advantages have made it a notable area of research. In our future work, we will concentrate on investigating the finite-time synchronization of discrete neural networks using event-triggered control methodologies.

**Conflicts of interest.** The authors declare no conflicts of interest.

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