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Polynomial synchronization of quaternion-valued memristive neural networks

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Abstract. This paper mainly utilizes the nondecomposition method to examine a problem of polynomial synchronization for the quaternion-valued memristive neural networks. First, to avoid considering the "magnitude" of a quaternion, the nonreduced-order and nondecomposition methods are applied to maintain computational efficiency without introducing new variables. Then, under a simple controller, the system's global polynomial synchronization is guaranteed by formulating a suitable Lyapunov functional and using quaternion properties. Finally, an illustrative numerical example is included to substantiate the conclusion.

Keywords: memristor, neural networks, quaternion, polynomial synchronization, nondecomposition.

1 Introduction

The memristor is the fourth basic circuit component after resistance, capacitance, and inductance, indicating the relationship between magnetic flux and charge, which was first proposed in [2]. Memristor neural network is a hardware neural network based on memristor, which is an important development direction of neuromorphic computing. As a passive device, memristor has nanoscale size and nonvolatility, which can realize the constant change of synaptic weight, build a highly integrated neural network structure, and make the artificial neural network have learning and memory ability and diversified functions.

Synchronization is a universal real-world phenomenon, meaning that two or more systems adjust to each other to achieve a common dynamic behavior. Synchronization

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is of great importance for its potential applications in many areas, including information processing, secure communication, biology systems, and so on. There have been many significant results on synchronization in [1, 5, 9, 10, 14]. Memristive neural networks are a type of nonlinear large-scale circuit system, and many synchronous control strategies have been developed to achieve specific goals or optimize performance, such as feedback control [6], adaptive control [8, 15, 22], sampled-data control [19], and event triggered control [12, 17, 18]. At present, feedback control is still the most common control strategy because it is easy to implement in hardware.

Very recently, a large number of conclusions have emerged regarding different types of synchronization problems for the memristive neural networks. Among them, polynomial synchronization is an important synchronization method, which is a type of decaying synchronization. Its convergence rate is no higher than that of global exponential synchronization, and lies between asymptotic synchronization and exponential synchronization. Moreover, it describes the convergence behavior of system trajectories over an unbounded state space. In recent years, polynomial synchronization has emerged as a significant research direction in neural network dynamics, achieving notable theoretical and practical advancements. [23, 24] concentrated on the application of the polynomial synchronization for the neural networks. Polynomial synchronization for the high-order neural networks are discussed in [3]. [20] and [21] discussed the polynomial synchronization of complex-valued and quaternion-valued neural networks, respectively.

Quaternions are extensively used in several fields including physics, applied mathematics, computer graphics, and control systems because of their notable and unique characteristics. Embedding quaternions into deep neural networks has attracted significant attention to neurocomputing researchers in recent years. Quaternion's algebra helps to reconstruct neural networks in the quaternionic domain. Quaternion-valued neural networks have also gained significant attention in the research community, with many studies [4, 11] investigating their properties and potential applications. The main advantage of quaternion-valued neural networks is their ability to represent and process rotational information more efficiently than real/complex-valued neural networks [7]. Quaternion's representation allows quaternion-valued neural networks to handle tasks effectively that involve rotation, such as image and video processing. On the other hand, it is also overcome the vanishing gradient problem [13] which is a common issue in traditional neural networks. This is because quaternions have a higher degree of freedom, which allows them to maintain their values even when the network is deep.

Strongly motivated by the above discussions, this paper is devoted to the investigation of the polynomial synchronization for quaternion-valued memristive neural networks. The main contributions of this paper are highlighted as follows:

(i) Based on the definition of the polynomial synchronization, the derived conclusions describe the convergence behavior of system trajectories over an unbounded state space.
(ii) Compared with the existing results, this article not only reduces the requirements for considering the "magnitude" of quaternions, but also provides more flexible criteria, which greatly improve the feasible of the obtained conclusions.

(iii) The proposed methodology provides tools for analyzing quaternion-valued memristive neural networks; moreover, the derived framework can also be applied to quaternion-valued neural networks with uncertain terms.

Notations. For $h = h^R + ih^I + jh^J + kh^K \in \mathbb{Q}$, $|h| = \sqrt{h^* h}$ is the norm of h.

2 Preliminaries

A type of memristive neural networks is presented as follows:

$$\begin{aligned}\dot{x}_p(t) = {}& -c_p x_p(t) + \sum_{q=1}^{n} a_{pq}\big(x_p(t)\big) f_q\big(x_q(t)\big) \\ & + \sum_{q=1}^{n} b_{pq}\big(x_p(t)\big) f_q\big(x_q(t-\tau)\big), \quad p = 1, \dots, n,\end{aligned} \tag{1}$$

where $c_p > 0$, $\tau > 0$ is referred to the transmission delay, $f_q(\cdot) \in \mathbb{Q}$ represents the neuron function, $a_{pq}(x_p(t)), b_{pq}(x_p(t)) \in \mathbb{Q}$ are defined as

$$a_{pq}\big(x_p(t)\big) = \begin{cases} a'_{pq} = a^R_{1pq} + a^I_{1pq} i + a^J_{1pq} j + a^K_{1pq} k, & |x_p(t)| \leqslant X_p, \\ a''_{pq} = a^R_{2pq} + a^I_{2pq} i + a^J_{2pq} j + a^K_{2pq} k, & |x_p(t)| > X_p, \end{cases}$$

$$b_{pq}\big(x_p(t)\big) = \begin{cases} b'_{pq} = b^R_{1pq} + b^I_{1pq} i + b^J_{1pq} j + b^K_{1pq} k, & |x_p(t)| \leqslant X_p, \\ b''_{pq} = b^R_{2pq} + b^I_{2pq} i + b^J_{2pq} j + b^K_{2pq} k, & |x_p(t)| > X_p. \end{cases}$$

Consider (1) as the drive system, then the corresponding response system is

$$\begin{aligned}\dot{\breve{x}}_p(t) = {}& -c_p \breve{x}_p(t) + \sum_{q=1}^{n} a_{pq}\big(\breve{x}_p(t)\big) f_q\big(\breve{x}_q(t)\big) \\ & + \sum_{q=1}^{n} b_{pq}\big(\breve{x}_p(t)\big) f_q\big(\breve{x}_q(t-\tau)\big) + u_p(t),\end{aligned} \tag{2}$$

where $u_p(t)$ is the controller to be designed.

Let $\theta_p(t) = \breve{x}_p(t) - x_p(t)$, then, based on systems (1) and (2), one has

$$\begin{aligned}\dot{\theta}_p(t) = {}& -c_p \theta_p(t) + \sum_{q=1}^{n} a_{pq}\big(\breve{x}_p(t)\big) g_q\big(\theta_q(t)\big) + \sum_{q=1}^{n} b_{pq}\big(\breve{x}_p(t)\big) g_q\big(\theta_q(t-\tau)\big) \\ & + \sum_{q=1}^{n} \big(a_{pq}\big(\breve{x}_p(t)\big) - a_{pq}\big(x_p(t)\big)\big) f_q\big(x_q(t)\big) + \sum_{q=1}^{n} \big(b_{pq}\big(\breve{x}_p(t)\big) \\ & - b_{pq}\big(x_p(t)\big)\big) f_q\big(x_q(t-\tau)\big) + u_p(t),\end{aligned}$$

where $g_q(\theta_q(t)) = f_q(\breve{x}_q(t)) - f_q(x_q(t))$.

The following assumption will be used throughout the paper.

(A) The function $f(\cdot)$ satisfies

$$\big|f_q(\delta_1) - f_q(\delta_2)\big| \leqslant l_q|\delta_1 - \delta_2|, \qquad \big|f_q(\delta)\big| \leqslant k_q,$$

where $l_q, k_q > 0$ are arbitrary constants.

Definition 1. (See [25].) Systems (1) and (2) are called globally polynomially synchronized if there exist constants $\tilde{M}, \lambda > 0$ such that

$$\big|\breve{x}(t) - x(t)\big| \leqslant \tilde{M}\left(\frac{1+t}{1+t_0}\right)^{-\lambda}$$

holds, where λ is the polynomial synchronous decay rate.

3 Main results

Theorem 1. *Under assumption* (A)*, if*

$$\eta \min_p \left[2(c_p + h_p) - \sum_{q=1}^{n}\left(\alpha_{pq}^2 + \beta_{pq}^2 + \mathcal{A}_{pq}^2 + \mathcal{B}_{pq}^2\right) - 2l_p^2 - 2\lambda\right] > 0,$$

$$M = 2\sum_{p=1}^{n} k_p^2$$

hold, where

$$\alpha_{pq} = \max\big\{|a'_{pq}|, |a''_{pq}|\big\}, \qquad \beta_{pq} = \max\big\{|b'_{pq}|, |b''_{pq}|\big\},$$
$$\mathcal{A}_{pq} = |a'_{pq} - a''_{pq}|, \qquad \mathcal{B}_{pq} = |b'_{pq} - b''_{pq}|,$$

then the global polynomial synchronization of systems (1) *and* (2) *can be derived under the following controller*:

$$u_p(t) = -h_p\theta_p(t), \quad h_p \in \mathbb{R}.$$

Proof. We construct the following Lyapunov function:

$$V_1(t) = \sum_{p=1}^{n}\big(\theta_p^*(t)\theta_p(t)\big)\left(\frac{1+t}{1+t_0}\right)^{2\lambda}.$$

Thus, calculating the Dini derivative of $V_1(t)$ yields

$$\begin{aligned} &D^+V_1(t) \\ &\quad= \sum_{p=1}^{n}\left\{\theta_p^*(t)\left[-c_p\theta_p(t) + \sum_{q=1}^{n} a_{pq}\big(\breve{x}_p(t)\big)g_q\big(\theta_q(t)\big)\right.\right. \\ &\qquad\left.\left. + \sum_{q=1}^{n} b_{pq}\big(\breve{x}_p(t)\big)g_q\big(\theta_q(t-\tau)\big) + \sum_{q=1}^{n}\big(a_{pq}\big(\breve{x}_p(t)\big) - a_{pq}(x_p(t))\big)f_q\big(x_q(t)\big)\right.\right. \end{aligned}$$

$$+\sum_{q=1}^{n}\big(b_{pq}\big(\breve{x}_p(t)\big)-b_{pq}\big(x_p(t)\big)\big)f_q\big(x_q(t-\tau)\big)+u_p(t)\Bigg]$$
$$+\Bigg[-c_p\theta_p^*(t)+\sum_{q=1}^{n}g_q^*\big(\theta_q(t)\big)a_{pq}^*\big(\breve{x}_p(t)\big)+\sum_{q=1}^{n}g_q^*\big(\theta_q(t-\tau)\big)b_{pq}^*\big(\breve{x}_p(t)\big)$$
$$+\sum_{q=1}^{n}f_q^*\big(x_q(t)\big)\big(a_{pq}(\breve{x}_p(t))-a_{pq}(x_p(t))\big)^*+\sum_{q=1}^{n}f_q^*(x_q\big(t-\tau)\big)\big(b_{pq}\big(\breve{x}_p(t)\big)$$
$$-b_{pq}\big(x_p(t)\big)\big)^*+u_p^*(t)\Bigg]\theta_p(t)+\theta_p^*(t)\theta_p(t)\frac{2\lambda}{1+t}\Bigg\}\left(\frac{1+t}{1+t_0}\right)^{2\lambda}.$$

From Lemma 2 in [16] and (A) one has

$$\begin{aligned}&\theta_p^*(t)a_{pq}\big(\breve{x}_p(t)\big)g_q\big(\theta_q(t)\big)+g_q^*(\theta_q(t))a_{pq}^*\big(\breve{x}_p(t)\big)\theta_p(t)\\&\quad\leqslant\big|a_{pq}\big(\breve{x}_p(t)\big)\big|^2\theta_p^*(t)\theta_p(t)+g_q^*\big(\theta_q(t)\big)g_q\big(\theta_q(t)\big)\\&\quad\leqslant\alpha_{pq}^2\theta_p^*(t)\theta_p(t)+l_q^2\theta_q^*(t)\theta_q(t),\end{aligned}\tag{3}$$

$$\begin{aligned}&\theta_p^*(t)b_{pq}\big(\breve{x}_p(t)\big)g_q\big(\theta_q(t-\tau)\big)+g_q^*\big(\theta_q(t-\tau)\big)b_{pq}^*\big(\breve{x}_p(t)\big)\theta_p(t)\\&\quad\leqslant\big|b_{pq}\big(\breve{x}_p(t)\big)\big|^2\theta_p^*(t)\theta_p(t)+g_q^*\big(\theta_q(t-\tau)\big)g_q\big(\theta_q(t-\tau)\big)\\&\quad\leqslant\beta_{pq}^2\theta_p^*(t)\theta_p(t)+l_q^2\theta_q^*(t-\tau)\theta_q(t-\tau),\end{aligned}\tag{4}$$

$$\begin{aligned}&\theta_p^*(t)\big(a_{pq}\big(\breve{x}_p(t)\big)-a_{pq}\big(x_p(t)\big)\big)f_q\big(x_q(t)\big)\\&\qquad+f_q^*\big(x_q(t)\big)\big(a_{pq}\big(\breve{x}_p(t)\big)-a_{pq}\big(x_p(t)\big)\big)^*\theta_p(t)\\&\quad\leqslant\mathcal{A}_{pq}^2\theta_p^*(t)\theta_p(t)+k_q^2,\end{aligned}$$

$$\begin{aligned}&\theta_p^*(t)\big(b_{pq}\big(\breve{x}_p(t)\big)-b_{pq}\big(x_p(t)\big)\big)f_q\big(x_q(t-\tau)\big)\\&\qquad+f_q^*\big(x_q(t-\tau)\big)\big(b_{pq}\big(\breve{x}_p(t)\big)-b_{pq}\big(x_p(t)\big)\big)^*\theta_p(t)\\&\quad\leqslant\mathcal{B}_{pq}^2\theta_p^*(t)\theta_p(t)+k_q^2.\end{aligned}$$

Considering the conclusions derived in (3) and (4), we have

$$\begin{aligned}D^+V_1(t)\leqslant\sum_{p=1}^{n}\Bigg\{&\Bigg[-2(c_p+h_p)+\sum_{q=1}^{n}\big(\alpha_{pq}^2+\beta_{pq}^2+\mathcal{A}_{pq}^2+\mathcal{B}_{pq}^2\big)+l_p^2+2\lambda\Bigg]\\&\times\big(\theta_p^*(t)\theta_p(t)\big)+l_p^2\theta_p^*(t-\tau)\theta_p(t-\tau)+2k_p^2\Bigg\}\left(\frac{1+t}{1+t_0}\right)^{2\lambda}.\end{aligned}$$

If $t\geqslant\tau$, then

$$\sum_{p=1}^{n}\theta_p^*(t)\theta_p(t)\leqslant\sup_{-\tau\leqslant s\leqslant0}\sum_{p=1}^{n}\big(\theta_p^*(s)\theta_p(s)\big)\mathrm{e}^{-\zeta t}$$

and

$$\sum_{p=1}^{n}\theta_p^*(t-\tau)\theta_p(t-\tau)\leqslant\sup_{-\tau\leqslant s\leqslant 0}\sum_{p=1}^{n}\big(\theta_p^*(s)\theta_p(s)\big)\mathrm{e}^{-\zeta(t-\tau)}.$$

Obviously,

$$\sum_{p=1}^{n}\big(\theta_p^*(t-\tau)\theta_p(t-\tau)\big)\leqslant\sup_{-\tau\leqslant s\leqslant 0}\sum_{p=1}^{n}\big(\theta_p^*(s)\theta_p(s)\big)$$

when $t<\tau$, that is,

$$\begin{aligned}D^+V_1(t)&\leqslant\sum_{p=1}^{n}\left\{\left[-2(c_p+h_p)+\sum_{q=1}^{n}\big(\alpha_{pq}^2+\beta_{pq}^2+\mathcal{A}_{pq}^2+\mathcal{B}_{pq}^2\big)+2l_p^2+2\lambda\right]\right.\\&\quad\left.\times\big(\theta_p^*(t)\theta_p(t)\big)+2k_p^2\right\}\left(\frac{1+t}{1+t_0}\right)^{2\lambda}\\&\triangleq-\eta V(t)+M\left(\frac{1+t}{1+t_0}\right)^{2\lambda},\end{aligned}$$

which signifies

$$V_1(t)\leqslant\mathrm{e}^{-\eta(t-t_0)}\left(V_1(t_0)-\frac{M}{\eta}\right)+\frac{M}{\eta}\leqslant V_1(t_0)-\frac{M}{\eta}+\frac{M}{\eta},$$

i.e.,

$$\sum_{p=1}^{n}\big(\theta_p^*(t)\theta_p(t)\big)\left(\frac{1+t}{1+t_0}\right)^{2\lambda}\leqslant V_1(t_0),$$

and this means that

$$\big|\theta(t)\big|\leqslant\sqrt{V_1(t_0)}\left(\frac{1+t}{1+t_0}\right)^{-\lambda}.$$

Hence, according to Definition 1, the global polynomial synchronization of systems (1) and (2) can be demonstrated. □

Theorem 2. *Under assumption* (A) *and* $|\theta_p(s)|\leqslant\delta$, *if*

$$\tilde{\eta}=\min_p\left[2(c_p+h_p)-\sum_{q=1}^{n}\big(\alpha_{pq}^2+\beta_{pq}^2+\mathcal{A}_{pq}^2+\mathcal{B}_{pq}^2\big)-l_p^2-1-2\lambda\right]>0,$$

$$l_p^2\leqslant 1,$$

hold, then systems (1) *and* (2) *can achieve global polynomial synchronization under the above controller.*

Proof. We choose the following Lyapunov functional:

$$V(t) = V_1(t) + V_2(t) \triangleq V_1(t) + \int_{t-\tau}^{t} \theta_p^*(s)\theta_p(s)\,\mathrm{d}s. \tag{5}$$

Subsequently, the differential of $V_2(t)$ is

$$D^+V_2(t) = \sum_{p=1}^{n} \theta_p^*(t)\theta_p(t) - \sum_{p=1}^{n} \theta_p^*(t-\tau)\theta_p(t-\tau). \tag{6}$$

Substituting (6) into (5), it follows that

$$\begin{aligned} D^+V(t) &\leqslant \sum_{p=1}^{n} \left\{ \left[-2(c_p + h_p) + \sum_{q=1}^{n} \left(\alpha_{pq}^2 + \beta_{pq}^2 + \mathcal{A}_{pq}^2 + \mathcal{B}_{pq}^2\right) + l_p^2 + 1 + 2\lambda \right] \right. \\ &\quad \left. \times \left(\theta_p^*(t)\theta_p(t)\right) + 2k_p^2 \right\} \left(\frac{1+t}{1+t_0}\right)^{2\lambda} \\ &\leqslant -\tilde{\eta}\left(\theta_p^*(t)\theta_p(t)\right)\left(\frac{1+t}{1+t_0}\right)^{2\lambda} + M\left(\frac{1+t}{1+t_0}\right)^{2\lambda} \\ &= -\tilde{\eta}V(t) + \tilde{\eta}V_2(t) + M\left(\frac{1+t}{1+t_0}\right)^{2\lambda} \\ &\leqslant -\tilde{\eta}V(t) + \tilde{\eta}\tau\delta^2 + M\left(\frac{1+t}{1+t_0}\right)^{2\lambda}. \end{aligned}$$

Set $S(t) = V(t) - \tau\delta^2$. Then

$$D^+S(t) \leqslant -\tilde{\eta}S(t) + M\left(\frac{1+t}{1+t_0}\right)^{2\lambda},$$

and this means that

$$V(t) \leqslant V(t_0),$$

i.e.,

$$\begin{aligned} \sum_{p=1}^{n} \left(\theta_p^*(t)\theta_p(t)\right)\left(\frac{1+t}{1+t_0}\right)^{2\lambda} &\leqslant \sum_{p=1}^{n} \left(\theta_p^*(t_0)\theta_p(t_0)\right) + \tau \sup_{t_0-\tau\leqslant s\leqslant t_0} \theta_p^*(s)\theta_p(s) \\ &\leqslant (1+\tau) \sup_{t_0-\tau\leqslant s\leqslant t_0} \sum_{p=1}^{n} \theta_p^*(s)\theta_p(s). \end{aligned}$$

Then one has

$$\left|\theta(t)\right| \leqslant \sqrt{1+\tau} \sup_{t_0-\tau\leqslant s\leqslant t_0} \left|\theta(s)\right| \left(\frac{1+t}{1+t_0}\right)^{-\lambda}.$$

Consequently, the global polynomial synchronization between systems (1) and (2) can be achieved by the designed controller. □

4 Numerical example

We consider system (1) with

$$a_{11}(\cdot)=\begin{cases}0.2+i+0.3j+0.2k,\\ 0.17+0.77i+0.25j+0.18k,\end{cases}$$

$$a_{12}(\cdot)=\begin{cases}0.3+0.2i+0.15j+0.2k,\\ 0.22+0.19i+0.33j+0.17k,\end{cases}$$

$$a_{21}(\cdot)=\begin{cases}0.23+0.18i+0.27j+0.35k,\\ 0.32+0.22i+0.16j+0.32k,\end{cases}$$

$$a_{22}(\cdot)=\begin{cases}0.8+0.2i+0.28j+0.77k,\\ 0.72+0.11i+0.29j+k,\end{cases}$$

$$b_{11}(\cdot)=\begin{cases}0.19+0.19i+j+0.6k,\\ 0.18+0.18i+0.8j+0.56k,\end{cases}$$

$$b_{12}(\cdot)=\begin{cases}0.33+0.21i+0.35j+0.22k,\\ 0.4+0.14i+0.24j+0.27k,\end{cases}$$

$$b_{21}(\cdot)=\begin{cases}0.1+0.12i+0.61j+0.4k,\\ 0.12+0.55i+0.56j+0.35k,\end{cases}$$

$$b_{22}(\cdot)=\begin{cases}0.2+0.17i+0.72j+0.13k,\\ 0.1+0.19i+0.66j+0.12k.\end{cases}$$

Moreover, let $X_p=1$, $c_1=0.89$, $c_2=0.89$, and $\tau=0.1$. The activation function is chosen as $f_q(\cdot)=0.1\sin(\cdot)$. In accordance with assumption (A), we have $l_q=0.1$ and $k_q=0.1$. Additionally, set $\lambda=0.1$ and $h_1=h_2=1$. Then, by Theorem 1, it follows that

$$\begin{aligned}&2(c_1+h_1)-\sum_{q=1}^{2}\left(\alpha_{1q}^2+\beta_{1q}^2+\mathcal{A}_{1q}^2+\mathcal{B}_{1q}^2\right)-2l_1^2-2\lambda\\&\quad=3.78-(1.17+0.1925+1.4322+0.3239+0.0567\\&\qquad+0.0398+0.0418+0.0244)-0.02-0.2\\&\quad=0.2787,\\&2(c_p+h_p)-\sum_{q=1}^{n}\left(\alpha_{pq}^2+\beta_{pq}^2+\mathcal{A}_{pq}^2+\mathcal{B}_{pq}^2\right)-2l_p^2-2\lambda\\&\quad=3.78-(0.2807+1.6146+0.5565+0.6042+0.0227\\&\qquad+0.0675+0.0103+0.0141)-0.02-0.2\\&\quad=0.3894.\end{aligned}$$

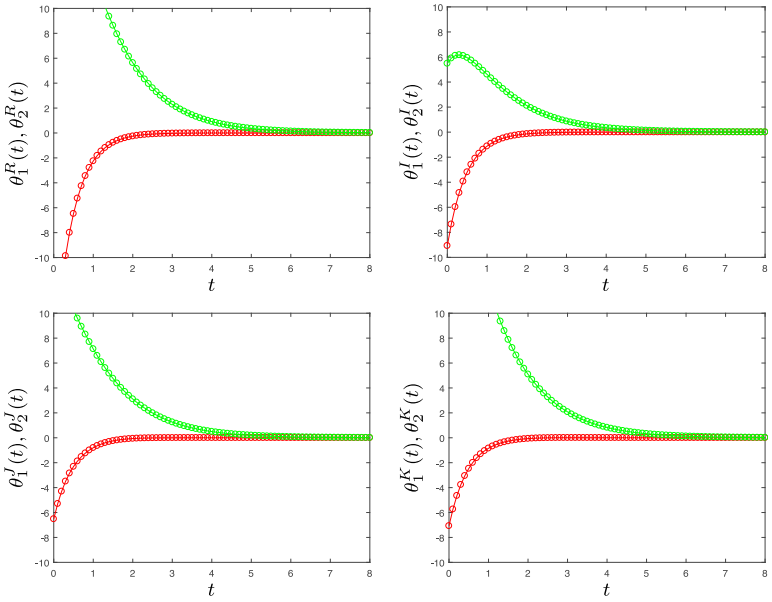


Figure 1. Time evolution of $\theta(t)$.

This implies that the global polynomial synchronization of systems (1) and (2) is achieved. Figure 1 illustrates the time evolution of $\theta(t)$.

5 Conclusion

In this paper, the polynomial synchronization problem of quaternion-valued memristive neural networks under a feedback controller has been investigated. First, to avoid considering the "magnitude" of a quaternion, the nonreduced-order and nondecomposition methods are applied to maintain computational efficiency without introducing new variables, which provide more flexible criteria. Then, under a simple controller, the system's global polynomial synchronization is guaranteed by formulating a suitable Lyapunov functional and using quaternion properties. Finally, an illustrative numerical example is included to substantiate the conclusion.

Although the quaternion norm is employed in this paper, whether other norms can be used remains an open question and will be investigated in future research.

Conflicts of interest. The authors declare no conflicts of interest.

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