

ENGEL ELASTICITIES DERIVED FROM LORENZ AND CONCENTRATION CURVES: CASE OF LATVIA

Ilze Balode

Faculty of Economics and Management Ventspils University College

Address: 101A Inženieru Street, Ventspils LV-3601, Latvia

E-mail: ilze.balode@venta.lv

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Abstract. Using information from a household budget survey of the Central Statistical Bureau of Latvia and with the help of the estimation of Lorenz curves of total consumption expenditure and consumption expenditure concentration curves of separate groups of goods, Engel elasticities, concentration indices, overall elasticity indices of twelve aggregated groups of goods are estimated. The results obtained are new and could be meaningfully interpreted in order to explain household behaviour.

Keywords: Lorenz curve, concentration curve, concentration index, overall elasticity index, Engel elasticity.

1. Introduction

The income elasticity of goods demand (Engel elasticity) is usually estimated exploiting the Engel function. Engel elasticities can also be estimated indirectly using Lorenz curves and concentration curves of the corresponding goods. P. C. Mahalanobis and N. S. Iyengar in 1960 widened the concept of the Lorenz curve, used the concept of a concentration curve and showed that the concentration index of a good is closely connected with its elasticity [6, 8]. Kakwani, Podder, Tran-Nam and others use this method to estimate expenditure elasticities [7, 9].

The connection between Engel elasticities and Lorenz curves and concentration curves of the corresponding goods is described in M. O. Haque's book [4].

The aim of the present paper is to study the behaviour of Latvia's households by estimating Engel elasticities and expenditure overall elasticity indices for twelve aggregated groups of goods.

The structure of the paper is as follows: first, notations are introduced and the method used is briefly described (Section 2); then, the data used in the study are described (Section 3); and, finally, the results of the study are presented (Section 4).

2. Engel elasticities derived from Lorenz and concentration curves

Let's look at inhabitant's consumption expenditure as a random variable X with value domain $[0, +\infty)$ and probability density $f(x)$.

The expected value of a random variable X is $\mu := E(X) = \int_0^{+\infty} x f(x) dx$.

The probability distribution function is

$$p := F(x) = \int_0^x f(u) du, \quad x \in [0, +\infty). \quad (1)$$

Let's denote the probability distribution $p(x)$ inverse function – quantile function as

$$x = x(p), \quad p \in [0, 1].$$

The function $\varphi = \varphi(x(p))$, where

$$\varphi(x) = [E(X)]^{-1} \cdot \int_0^x u f(u) du, \quad (2)$$

where $x \in [0, +\infty)$, is called the Lorenz function [4].

Equations $p = p(x)$, $\varphi = \varphi(x)$, where $x \in [0, +\infty)$, define the dependence of the variable $\varphi = \varphi(x)$ on the variable $p = p(x)$ via parameter x .

Equations $p = p(x)$, $\varphi = \varphi(x)$, where $x \in [0, +\infty)$, are parametric equations of the Lorenz curve.

Using the Lorenz curve, the Gini index $G(X)$ can be calculated:

$$G(X) = 1 - 2 \int_0^{+\infty} \varphi(x) f(x) dx. \quad (3)$$

Gini index, $G(X) \in [0, 1]$, is a measure of the inequality of a distribution. If the value of Gini index is zero, this corresponds to perfect equality, while if the value of Gini index is one, this corresponds to perfect inequality.

Alongside the random variable X , let's look also at a random variable $q(X)$ – stochastic consumption expenditure for the purchase of a definite commodity.

The expected value of the random variable $q(X)$ is $\mu_q := E(q(X)) = \int_0^{+\infty} q(x) f(x) dx$.

The function $\psi = \psi(x(p))$, where

$$\psi(x) = [E(q(X))]^{-1} \cdot \int_0^x q(u) f(u) du, \quad (4)$$

where $x \in [0, +\infty)$, is called the concentration curve of the given commodity [4].

Equations $p = p(x)$, $\psi = \psi(x)$, where $x \in [0, +\infty)$, define the dependence of the variable $\psi = \psi(x)$ on the variable $p = p(x)$ through the parameter x .

Equations $p = p(x)$, $\psi = \psi(x)$, where $x \in [0, +\infty)$, are parametric equations of the concentration curve of a given commodity.

Using the concentration curve of a commodity, the concentration index $C(q(X); X)$ of the given commodity can be calculated:

$$C(q(X); X) = 1 - 2 \int_0^{+\infty} \psi(x) f(x) dx. \quad (5)$$

The overall elasticity index of the given commodity is defined as a difference [12]:

$$E_q := C(q(X); X) - G(X). \quad (6)$$

The commodity is classified as elastic if the overall elasticity index is negative; the commodity is classified as inelastic if the overall elasticity index is positive [12].

Let's calculate the first and the second derivatives φ_p, φ_{pp} of the Lorenz function $\varphi = \varphi(x(p))$.

Let's calculate the first and the second derivatives ψ_p, ψ_{pp} of the concentration function $\psi = \psi(x(p))$.

The derivatives obtained allow calculating Engel elasticity of the commodity $q^{el}(x)$:

$$q^{el}(x) = \frac{\psi_{pp}}{\psi_p} \cdot \frac{\varphi_p}{\varphi_{pp}}. \quad (7)$$

We assume that the derivatives used exist and fraction denominators are not equal to zero.

Negative Engel elasticity is associated with inferior goods; positive elasticity is associated with normal goods. Engel elasticities allow classifying normal commodities as necessity and luxury commodities. If Engel elasticities are less than one, the given groups are classified as necessity goods. If Engel elasticities are greater than one, the given groups are classified as luxury goods.

Let's assume that the total consumption expenditure X consists of consumption expenditure for n commodities:

$$X = q_1(X) + q_2(X) + \dots + q_n(X). \quad (8)$$

The expected value of a random variable $q_i(X)$ is denoted μ_i ; $i = 1, 2, \dots, n$.

The corresponding overall commodity elasticities are denoted

$$E_i := C(q_i(X); X) - G(X); i = 1, 2, \dots, n. \quad (9)$$

The equation holds [12]:

$$\mu^{-1} (\mu_1 E_1 + \mu_2 E_2 + \dots + \mu_n E_n) = 0. \quad (10)$$

3. Data

Information for the study: household budget survey (HBS) 2005 by the Central Statistical Bureau of Latvia.

“In the HBS, households are selected by a two-stage stratified random sampling procedure. Primary sampling units (PSUs) are the population census enumeration areas and are selected with probabilities proportional to the size. At the second stage, within each PSU, households are selected with a simple random sampling procedure.” [5]

In 2005, 3774 households were surveyed. When estimating Lorenz curves and concentration curves, one household whose consumption expenditure was negative, one household whose disposable income was negative and one household whose income was zero were excluded.

In the HBS, consumption expenditure per household member per month, on average, is used.

In the HBS, the Classification of Individual Consumption by Purpose (COICOP) is used [3], where all household consumption expenditure at the first level is distributed into 12 consumption groups according to its functional meaning.

First-level consumption expenditure group codes and titles are presented in Table 1.

Table 1. Consumption expenditure group codes and titles

Consumption expenditure codes	Consumption expenditure titles
01	Food and non-alcoholic beverages
02	Alcoholic beverages, tobacco
03	Clothing and footwear
04	Housing, water, electricity, gas and other fuels
05	Furnishings, household equipment and routine household maintenance
06	Health
07	Transport
08	Communications
09	Recreation and culture
10	Education
11	Restaurants, cafes and hotels
12	Miscellaneous goods and services

In 2005, the largest household consumption expenditure groups were expenditure on food and non-alcoholic beverages (31%) and expenditure on housing, water, electricity, gas and other fuels (12%). Taken together, the two expenditure groups make up 43% of total household consumption expenditure.

Calculations were performed applying the following software: SAS, SPSS, Microsoft Excel.

4. Results

Using an empirical Lorenz curve for total consumption expenditure and an empirical Lorenz and concentration curves of twelve consumption expenditure groups, Gini index of total consumption expenditure and Gini indices for separate consumption expenditure groups, the concentration indices of consumption expenditure groups and overall elasticities indices were estimated (Table 2). In Figure 1, Lorenz curve for total consumption expenditure and concentration curves for two consumption expenditure groups (01 – food and non-alcoholic beverages, and 03 – clothing and footwear) are shown.

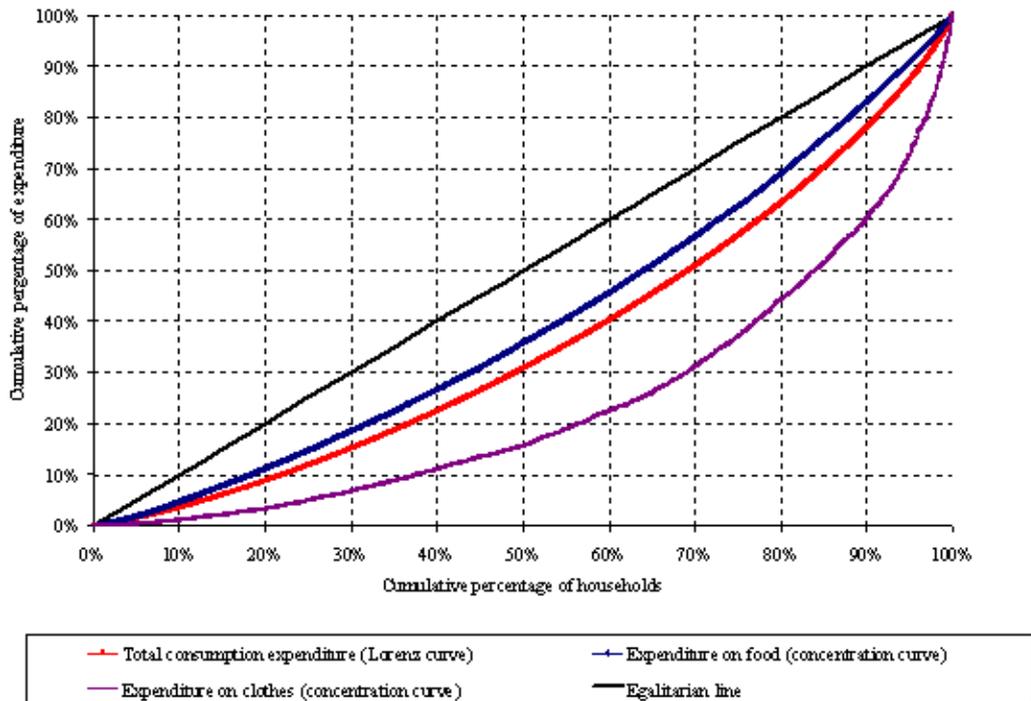


Fig. 1. Lorenz curve of total consumption expenditure, concentration curves of food consumption expenditure and clothing consumption expenditure of Latvia's households in 2005

It is seen in Figure 1 that the concentration curve of food consumption expenditure is located above the Lorenz curve of total consumption expenditure, but the concentration curve of clothing consumption expenditure is located below the Lorenz curve of total consumption expenditure. The concentration index for food is 0.20, the concentration index for clothing is 0.52.

Concentration curves are widely interpreted. For a normal commodity, the concentration curve is under the egalitarian line, for an inferior commodity – above the egalitarian line, for a necessity commodity – between the egalitarian line and Lorenz curve, for a luxury commodity – under the Lorenz curve.

For example, the graph of the clothing concentration curve shows that the expenditure on clothing of 10% of rich households (according to total consumption expenditure per household member) make up 40% of the total consumption expenditure on clothing of all households, while the expenditure on clothing of 10% of the poorest households constitutes only 2% of the total consumption expenditure on clothing of all households.

Respectively, the overall elasticity index of estimated food consumption expenditure is negative, but the overall elasticity index of clothing consumption expenditure is positive. Hence, for all households in general, food is classified as a normal inelastic commodity, while clothing – as a normal elastic commodity.

Table 2. Gini index, concentration index, and overall expenditure elasticity index for consumption expenditure groups of Latvia's households in 2005

	Gini index	Consumption expenditure*	Concentration index	Overall elasticity index
01 – Food and non-alcoholic beverages	0.28	39.65	0.20	-0.15
02 – Alcoholic beverages, tobacco	0.71	4.22	0.32	-0.03
03 – Clothing and footwear	0.73	9.98	0.52	0.17
04 – Housing, water, electricity, gas and other fuels	0.49	15.42	0.29	-0.06
05 – Furnishings, household equipment and routine household maintenance	0.70	7.01	0.48	0.13
06 – Health	0.72	5.06	0.31	-0.04
07 – Transport	0.72	14.81	0.55	0.20
08 – Communications	0.51	7.82	0.00	-0.35
09 – Recreation and culture	0.66	8.55	0.50	0.15

	Gini index	Consumption expenditure*	Concentration index	Overall elasticity index
10 – Education	0.93	1.86	0.49	0.14
11 – Restaurants, cafes and hotels	0.78	7.15	0.54	0.19
12 – Miscellaneous goods and services	0.63	6.52	0.49	0.14
Consumption expenditure	0.35	128.05		

* Average per household member per month, LVL.

The estimations prove that consumption expenditure groups “03 – clothing and footwear”, “05 – furnishings, household equipment and routine household maintenance”, “07 – transport”, “09 – recreation and culture”, “10 – education”, “11 – restaurants, cafes and hotels”, “12 – miscellaneous goods and services” are classified as elastic, while consumption expenditure groups “01 – food and non-alcoholic beverages”, “02 – alcoholic beverages, tobacco”, “04 – housing, water, electricity, gas and other fuels”, “06 – health”, “08 – communication” – as inelastic.

All twelve consumption expenditure groups are classified as normal.

Overall expenditure elasticity indices in Table 2 fulfil the equation (10).

To calculate Engel elasticities for consumption expenditure groups, it is necessary to estimate Lorenz curves and concentration curves. The adequate choice of the functional form of the Lorenz curve and the concentration curve is very important because it will be used to estimate income elasticities. The author deals with three functional forms:

$$\varphi(p) = \left(1 - (1-p)^\alpha\right)^\beta, \text{ where } 0 < \alpha, \beta \geq 1 \quad (\text{Rashe } et al., 1980) [11],$$

$$\varphi(p) = p - \alpha p^\alpha (1-p)^\beta, \text{ where } 0 < \alpha \leq 1, 0 < \beta \leq 1 \quad (\text{Kakwani, 1986}) [8],$$

$$\varphi(p) = p^\alpha [1 - (1-p)^\beta], \text{ where } 0 \leq \alpha, 0 < \beta \leq 1 \quad (\text{Ortega } et al., 1991) [10],$$

where p is a cumulative relative number of households and $\varphi(p)$ is cumulative relative consumption expenditure of households.

Estimating Lorenz curves in the form suggested by Kakwani (1986) and Ortega (1991) in their works, compared to the Lorenz curve functional form offered by Rashe (1980), better estimations are acquired in the latter case as a comparative criterion using the residual sum of squares and a coefficient of determination. If compared only by the coefficient of determination, both curve forms offered by the authors Rashe (1980) and Ortega (1991) should be recognized as equally good. Further in the estimations, the functional form by Rashe (1980) is used.

In Table 3, the estimation of the Lorenz curve for total consumption expenditure, the estimation of concentration curves for twelve consumption expenditure groups and Engel elasticities for consumption expenditure groups at average household total consumption expenditure calculated with their help are given.

Table 3. Estimates of parameters of Lorenz curves and concentration curves, Engel elasticities for consumption expenditure groups of Latvia’s households in 2005

	Parameter α	Parameter β	Coefficient of determination	Engel elasticity
01 – Food and non-alcoholic beverages	0.8538 (0.0004)	1.2680 (0.0007)	0.9999	0.546
02 – Alcoholic beverages, tobacco	0.6248 (0.0011)	1.1871 (0.0020)	0.9984	0.892
03 – Clothing and footwear	0.5877 (0.0006)	1.6844 (0.0020)	0.9993	1.584
04 – Housing, water, electricity, gas and other fuels	0.8043 (0.0005)	1.4153 (0.0010)	0.9998	0.804
05 – Furnishings, household equipment and routine household maintenance	0.6288 (0.0005)	1.6565 (0.0016)	0.9996	1.458
06 – Health	0.8402 (0.0011)	1.5327 (0.0023)	0.9992	0.858
07 – Transport	0.5893 (0.0009)	1.7870 (0.0032)	0.9985	1.709
08 – Communications	1.0004 (0.0000)	1.7303 (0.0000)	1.0000	0.717
09 – Recreation and culture	0.5527 (0.0003)	1.5021 (0.0081)	0.9998	1.431
10 – Education	0.9752 (0.0023)	23.8052 (0.1737)	0.9962	1.557
11 – Restaurants, cafes and hotels	0.5766 (0.0011)	1.7303 (0.0035)	0.9980	1.667
12 – Miscellaneous goods and services	0.5579 (0.0004)	1.4973 (0.0012)	0.9997	1.414
Consumption expenditure	0.6617 (0.0002)	1.2678 (0.0004)	1.0000	

Note: in parentheses – the standard error.

For consumption expenditure groups “01 – food and non-alcoholic beverages”, “02 – alcoholic beverages and tobacco”, “04 – housing, water, electricity, gas and other fuels”, “06 – health”, “08 – communication”, Engel elasticities

are less than one; therefore, the given groups are classified as necessity goods. For consumption expenditure groups “03 – clothing and footwear”, “05 – furnishings, household equipment and routine household maintenance”, “07 – transport”, “09 – recreation and culture”, “10 – education”, “11 – restaurants, cafes and hotels”, “12 – miscellaneous goods and services”, Engel elasticities are greater than one; therefore, the given groups are classified as luxury goods.

This classification of goods equals the classification according to overall elasticity indices (Table 2).

In the research, Engel elasticities in household income quintiles at corresponding consumption expenditure average values are estimated (Table 4). A quintile is one-fifth of the number of the households surveyed that are arranged in ascending order according to income per household member [5].

Table 4. Engel elasticities of consumption expenditure groups in household income quintiles of Latvia’s households in 2005

	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
01 – Food and non-alcoholic beverages	0.668	0.607	0.573	0.524	0.467
02 – Alcoholic beverages, tobacco	0.661	0.774	0.838	0.937	1.070
03 – Clothing and footwear	1.789	1.691	1.634	1.542	1.405
04 – Housing, water, electricity, gas and other fuels	1.021	0.914	0.854	0.764	0.653
05 – Furnishings, household equipment and routine household maintenance	1.692	1.579	1.514	1.411	1.265
06 – Health	1.244	1.053	0.947	0.786	0.587
07 – Transport	2.015	1.868	1.783	1.646	1.448
08 – Communications	1.524	1.123	0.901	0.569	0.173
09 – Recreation and culture	1.414	1.424	1.428	1.432	1.425
10 – Education	2.139	1.855	1.694	1.444	1.111
11 – Restaurants, cafes and hotels	1.901	1.789	1.724	1.619	1.461
12 – Miscellaneous goods and services	1.399	1.408	1.411	1.414	1.407
Average consumption expenditure per month, LVL	68.05	94.45	111.21	145.19	244.59

It is obvious that the consumption expenditure group “01 – food and non-alcoholic beverages” elasticity in all household quintile groups is less than one, and it decreases as household income increases.

For consumption expenditure groups “03 – clothing and footwear”, “05 – furnishings, household equipment and routine household maintenance”, “07 – transport”, “10 – education”, “11 – restaurants, cafes and hotels”, expenditure elasticity in all quintile groups is greater than one, and it decreases as household income increases.

For consumption expenditure groups “09 – recreation and culture” and “12 – miscellaneous goods and services”, expenditure elasticities in all quintile groups after rounding is 1.4.

For consumption expenditure group “02 – alcoholic beverages and tobacco”, consumption expenditure increases as household income increases; for the fifth quintile households, elasticity is greater than one.

For consumption expenditure group “04 – housing, water, electricity, gas and other fuels”, elasticity decreases as household income increases; for the first quintile households, it is greater than one.

For consumption expenditure groups “06 – health” and “08 – communication”, expenditure elasticity decreases as household income increases; for the first and second household quintile groups, expenditure elasticity is greater than one.

The Engel elasticities obtained in the present paper are not inconsistent with those acquired in paper [1], which were estimated with the help of the Linear Expenditure System (LES) and the Almost Ideal Demand System (AIDS).

5. Conclusions

1. The Engel elasticities estimated with the help of Lorenz curves and concentration curves of the corresponding commodity do not contradict to the results obtained using the demand systems. The graphical instruments of the method provide comprehensive information about household inequality in the field of consumption expenditure on different goods.

2. Engel elasticities and overall elasticity indices allow equally classify twelve consumption expenditure groups as well as draw the same conclusions about household behaviour in relation to the consumption of different goods according to changes in the total consumption expenditure. Consumption expenditure groups “01 – food and non-alcoholic

beverages”, “02 – alcoholic beverages and tobacco”, “04 – housing, water, electricity, gas and other fuels”, “06 – health”, “08 – communication” are classified as necessity goods, while consumption expenditure groups “03 – clothing and footwear”, “05 – furnishings, household equipment and routine household maintenance”, “07 – transport”, “09 – recreation and culture”, “10 – education”, “11 – restaurants, cafes and hotels”, “12 – miscellaneous goods and services” are classified as luxury goods.

3. Studying Engel elasticities in quintile groups, it was concluded that the consumption expenditure group “01 – food and non-alcoholic beverages” in all household quintile groups has elasticity less than one, and it decreases as household income increases. For consumption expenditure groups “03 – clothing and footwear”, “05 – furnishings, household equipment and routine household maintenance”, “07 – transport”, “10 – education”, “11 – restaurants, cafes and hotels”, expenditure elasticity in all quintile groups is greater than one, and it decreases as household income increases. For consumption expenditure group “04 – housing, water, electricity, gas and other fuels”, expenditure elasticity decreases as income increases; for the first quintile households, it is greater than one.

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ENGELIO ELASTINGUMAS APIBRĒŽTAS NAUDOJANT LORENCO IR KONCENTRACIJOS KREIVES

Ilze Balode

Santrauka. Remiantis Latvijos statistikos Namų ūkių biudžetų tyrimo duomenimis, Engelio elastingumas, koncentracijos indeksai, bendri elastingumo indeksai pagal 12 agreguotų prekių / paslaugų grupių yra įvertinti, naudojant bendrą vartojimo išlaidų Lorenco kreivės ir atskirų grupių prekių / paslaugų vartojimo išlaidų koncentracijos kreivių įvertinimą. Gauti rezultatai yra nauji ir gali būtų prasmingai interpretuojami, siekiant įvertinti namų ūkių padėtį.

Reikšminiai žodžiai: Lorenco kreivė, koncentracijos kreivė, bendras elastingumo indeksas, Engelio elastingumas.