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LEE–CARTER MORTALITY FORECASTING

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Abstract. In this paper, we focus on Lee–Carter mortality forecasting. In this paper, we focus on Lee–Carter mortality forecasting. In this paper, we focus on Lee–Carter mortality forecasting. In this paper, we focus on Lee–Carter mortality forecasting. In this paper, we focus on Lee–Carter mortality forecasting. In this paper, we focus on Lee–Carter mortality forecasting.

Key words : word1, word2, word3, word4.

1. Introduction

Mortality is not constant over time; moreover, it changes differently in different age groups. Therefore, it is important to identify the mortality trend and be able to predict mortality rates accurately.

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2. Lee–Carter model

Suppose that $m_{x,t}$ is the death rate for age x in year t, i.e. the ratio between the total number of deaths in the population of age x in year t and the total population of age x in year t $(N_{x,t})$:

$$m_{x,t} = \frac{D_{x,t}}{N_{x,t}},$$

and $\mu_{x,t} = \ln(m_{x,t})$ – empirical force of mortality. Lee and Carter [5] suggested a linear form for the force of mortality $\mu_{x,t}$:

$$\mu_{x,t} = \ln(m_{x,t}) = \alpha_x + \beta_x k_t + \varepsilon_{x,t},\ x = 1,\dots,A; t = 1,\dots,T, \tag{1}$$

where α_x, β_x are age-specific parameters, k_t – time-specific parameter, and $\varepsilon_{x,t}$ – independent identically distributed Gaussian errors $N(0,\sigma^2)$. Parameters α_x show the general rate of mortality for a certain age, and parameters k_t – the general rate of mortality for a certain time. It can be easily proved that the expression (1) of the force of mortality $\mu_{x,t}$ is invariant with respect to the transformations:

$$(\beta_x, k_t) \to (c\beta_x, k_t/c);\ (\alpha_x, k_t) \to (\alpha_x - c\beta_x, k_t + c) \text{ for some } c \in \mathbb{R}\backslash\{0\}.$$

So we can require the parameters β_x, k_t to satisfy these conditions:

$$\sum_{x=1}^{A} \beta_x = 1;\ \sum_{t=1}^{T} k_t = 0. \tag{2}$$

These conditions ensure the unambiguousness of parameters β_x and k_t.

Given the restriction of $\sum_{t=1}^{T} k_t = 0$, parameters α_x, $x = 1,\ldots,A$, are estimated by averages of the force of mortality over a time period, i.e.

$$\tilde{\alpha}_x = \frac{1}{T}\sum_{t=1}^{T}\mu_{x,t}, \text{ for all } x = 1,\ldots,A. \tag{3}$$

Then random variables $\mu_{x,t} - \tilde{\alpha}_x = \beta_x k_t + \varepsilon_{x,t}$, $x = 1,\ldots,A$; $t = 1,\ldots,T$, are Gaussian $N(\beta_x k_t, \sigma^2)$. According to [1] and [5], the optimal method to find the estimators of parameters β_x and k_t is SVD (singular value decomposition) of the matrix of variables $z_{x,t} = \mu_{x,t} - \tilde{\alpha}_x$, $x = 1,\ldots,A$; $t = 1,\ldots,T$.

Given the matrix $\mathbf{Z} = (z_{x,t})_{x=1,\ldots,A;\, t=1,\ldots,T}$, we can compute normalized eigenvectors $\mathbf{u}_1 = (u_{1,1},\ldots,u_{1,T})^T$ and $\mathbf{v}_1 = (v_{1,1},\ldots,v_{1,A})^T$ of the matrices $\mathbf{Z}^T\mathbf{Z}$ and $\mathbf{Z}\mathbf{Z}^T$, which correspond to the largest eigenvalue λ_1. Then the estimators of β_x, $x = 1,\ldots,A$, which satisfy the conditions (2) and estimators of k_t, $t = 1,\ldots,T$, are:

3. Empirical data analysis

Mortality data, population size and the number of deaths in Lithuania, France and Belarus are taken from the Berkeley Human Mortality Database, University of California (www.mortality.org). Lithuanian data is available for the period from 1959 to 2010, for ages 0 to 110 years, for men and women separately and together.

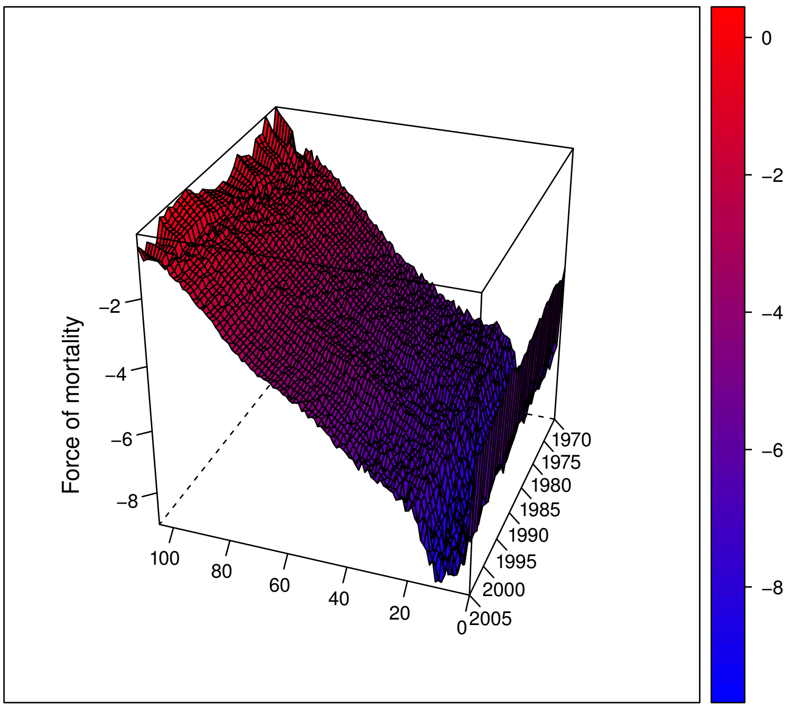

Figure 1: Empirical force of mortality of the Lithuanian population

We use the data from 1970 for our calculations because the data until 1970 might be unreliable. From Figure 1, where the empirical force of mortality of the Lithuanian population is shown, we can see that the optimal age interval for modelling is 20 to 90 years. Analogous surfaces are similar for France and Belarus, therefore, for these countries, we use the same time and age intervals. Later, the two data subsets will be taken for men and women separately:

- *Subset 1*: 1970 to 2005, 20 to 90 years,
- *Subset 2*: 2006 to 2010 (2009 for France), 20 to 90 years.

From the first subset, the parameters of the model are estimated. From the second subset, we can compare the models and the estimates of the force of mortality with the empirical data.

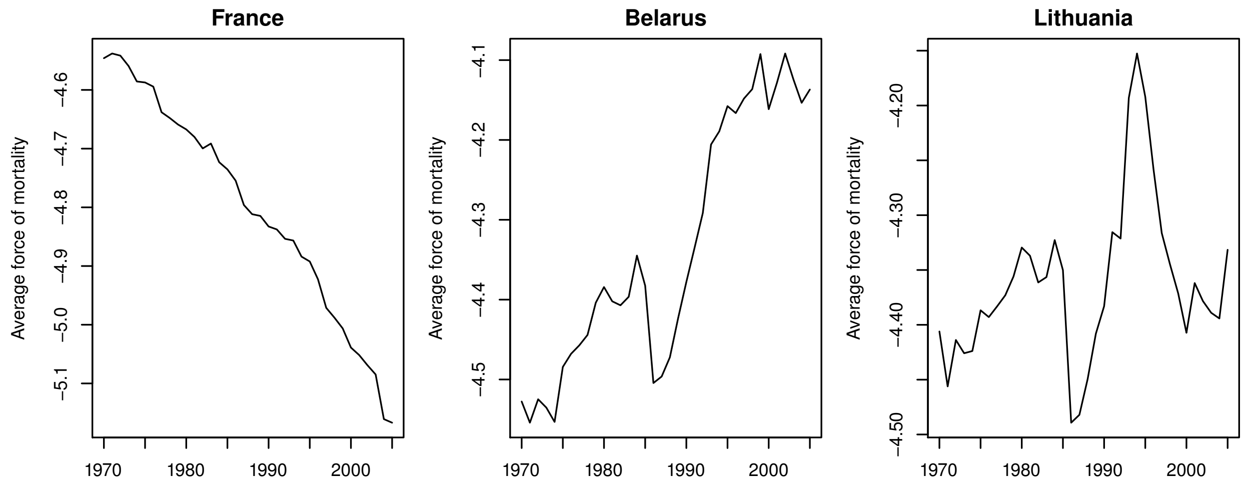

Figure 2: Average empirical force of mortality

Figure 2 plots the average empirical force of mortality $\bar{\mu}_t, t = 1970, ..., 2005$, $\left(\bar{\mu}_t = \frac{1}{71}\sum_{x=20}^{90}\mu_{x,t}\right)$ for France, Belarus and Lithuania for the period 1970 to 2005. The mortality of the French population is characterised by a significant dawnward trend; the average empirical force of mortality of Belarus increases over time, but with some fluctuations. Meanwhile, the average empirical force of mortality of Lithuania fluctuates without any visible trend.

4. $\mu_{x,t}$ forecast for Lithuania

4.1. Classical model

We calculate $\hat{\alpha}_x$, $\hat{\beta}_x$, $x = 20, ..., 90$, and $\hat{k}_t$, $t = 1970, ..., 2005$, for Lithuanian data. The estimates are shown in Figure **??**. The p-values of the ADF test are 0.4711 for men and 0.4377 for women.

Hence we model $\hat{k}_t$ as a random walk with a drift....

4.2. $\hat{\mathbf{k}}_\mathbf{t}$ with one and two lags

Let us consider that $\hat{k}_t$ is a random process with one lag, i.e. a second-order autoregressive process with a drift.

Table 1: The characteristics of the errors.

		$\hat{\sigma}_{rw}$	p-value of K–S test	p-value of χ^2 test
Men	$\hat{k}1_t$	3.0952	0.3743	0.3851
	$\hat{k}2_t$	0.6068	0.02	0.0415
Women	$\hat{k}1_t$	3.1739	0.1655	0.5438
	$\hat{k}2_t$	0.7497	0.0074	0.0351

Let us consider that $\hat{k}_t$ is a random process with one lag, i.e. a second-order autoregressive process with a drift

5. Conclusions

While comparing the suitability of the Lee–Carter model for different countries, we have obtained that the model most accurately describes and predicts mortality for Country.

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STRAIPSNIO PAVADINIMAS

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Santrauka. Darbe nagrinėjamas Lee–Carter'io metodas mirtingumui prognozuoti. Analizuojamos modelio liekanos bei mirtingumo tendencijos. Darbe nagrinėjamas Lee–Carter'io metodas mirtingumui prognozuoti. Analizuojamos modelio liekanos bei mirtingumo tendencijos.

Reikšminiai žodžiai: zodis1, zodis2, zodis3, zodis4